Indiana University Bloomington

THE COLLEGE OF ARTS + SCIENCES
Department of Physics

National Nuclear Physics Summer School 2024

July 15 – July 26, 2024
Bloomington, IN

The Electron-Ion Collider (EIC)

- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons with polarized beam(s)
  – *Spin as another knob*
- Lec. 4: Probing Structure of Hadrons without breaking them? Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

Jianwei Qiu

Theory Center, Jefferson Lab

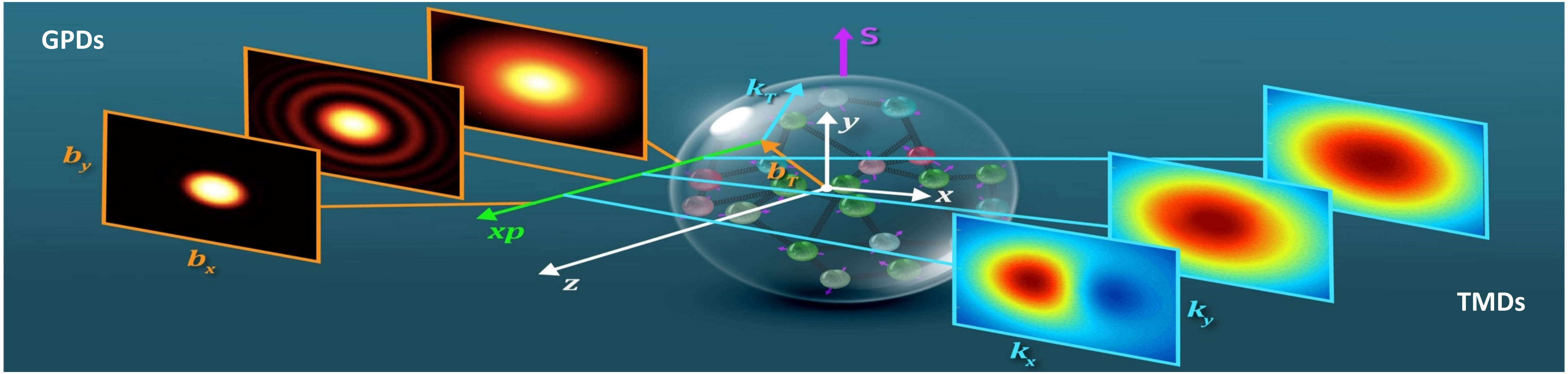
Jefferson Lab

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

How to Explore Internal Structure of a Hadron without Breaking it?

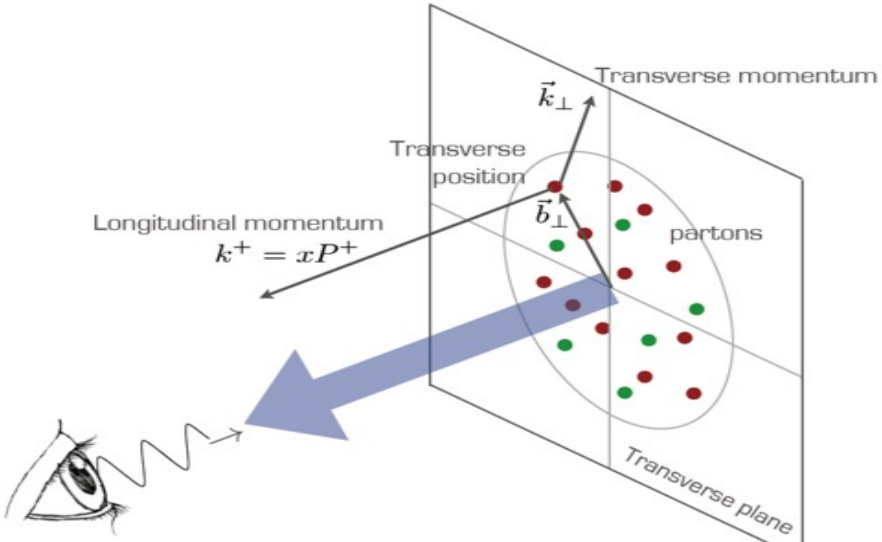
❑ 3D hadron structure:



❑ Need new observables with two distinctive scales:

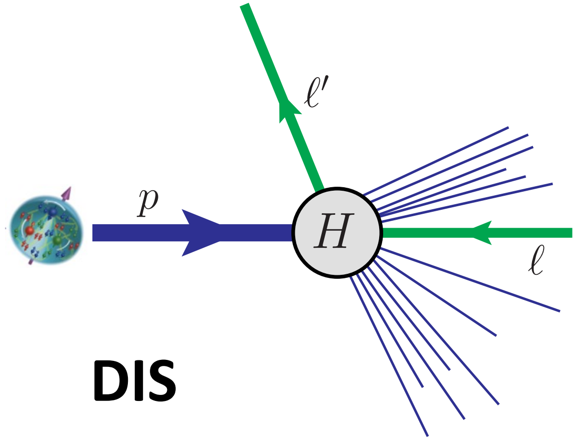
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\mathrm{QCD}}$$

- **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
- **"Soft" scale:** Q_2 **to be more sensitive to the emergent regime of hadron structure ~ 1/fm**

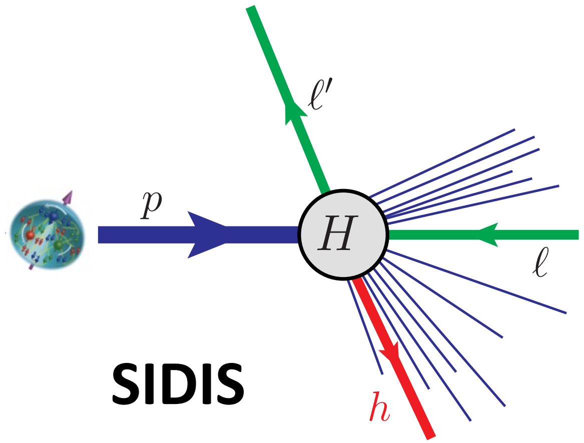


Inclusive vs. Exclusive – Partonic structure without breaking the hadron!

Inclusive scattering

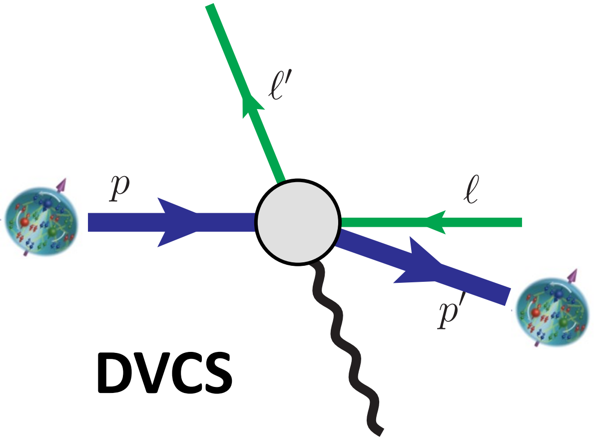


DIS



SIDIS

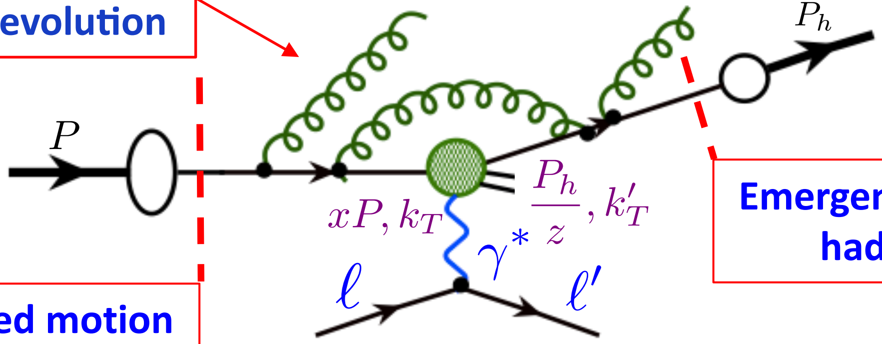
Exclusive diffraction



DVCS

Gluon shower – QCD evolution

Confined motion



Emergence of a hadron hadronization

$$e + P \to e + h + X$$

Inclusive vs. Exclusive – Partonic structure without breaking the hadron!

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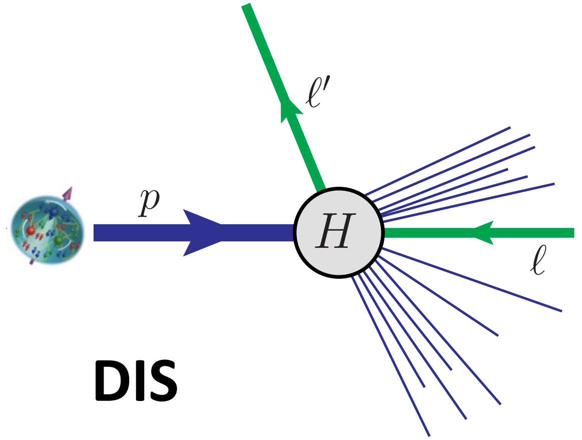

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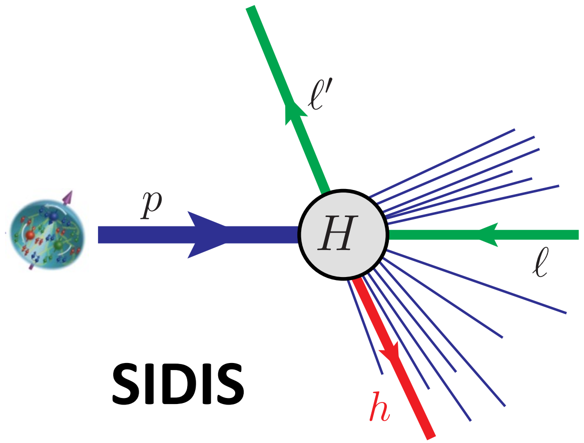

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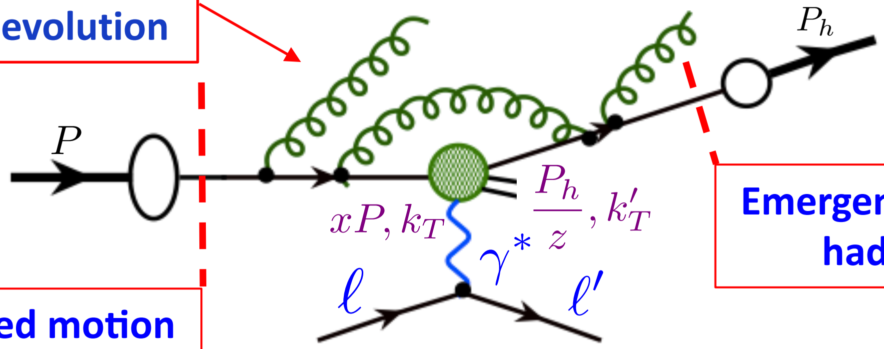

Emergence of a hadron
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$$e + P \to e + h + X$$

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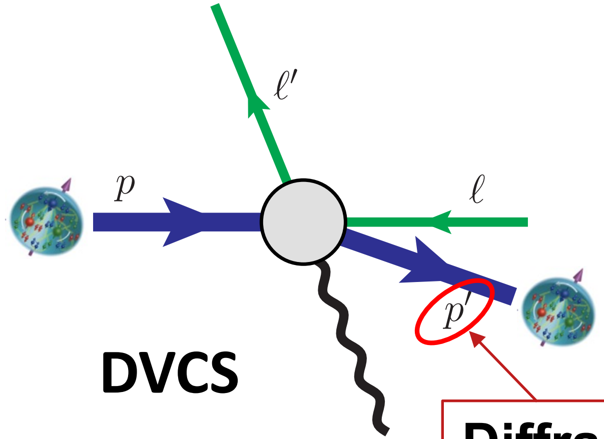


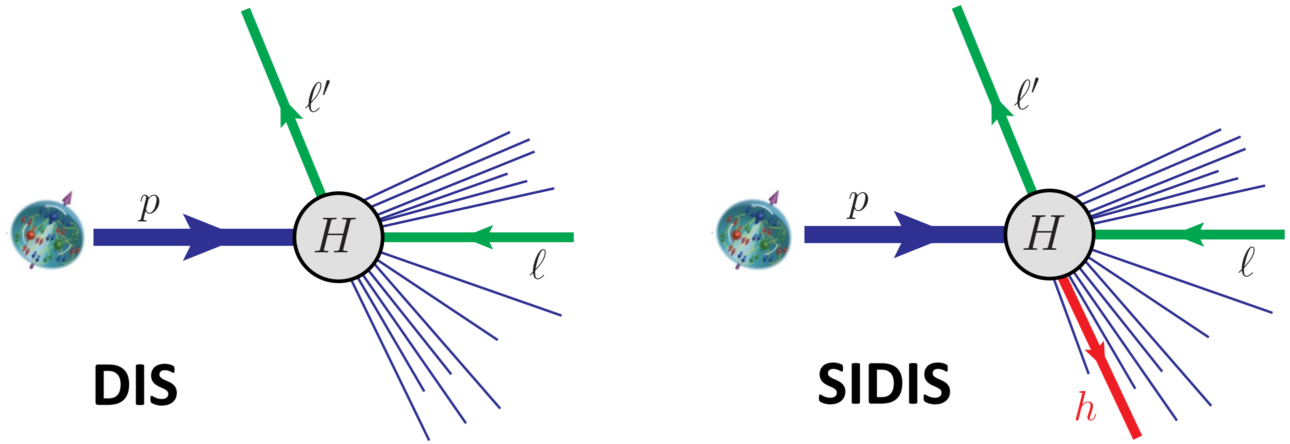
DVCS

Diffraction

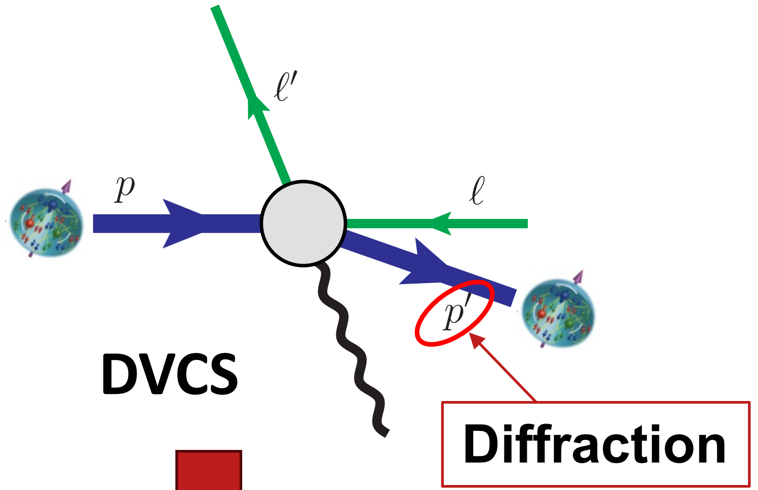
$$Q^2 = -(\ell - \ell')^2 \gg -(p - p')^2 = -t$$

Inclusive vs. Exclusive – Partonic structure without breaking the hadron!

Inclusive scattering



Exclusive diffraction



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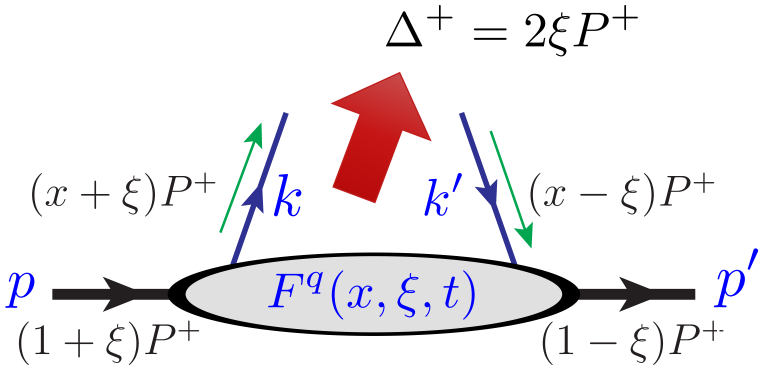
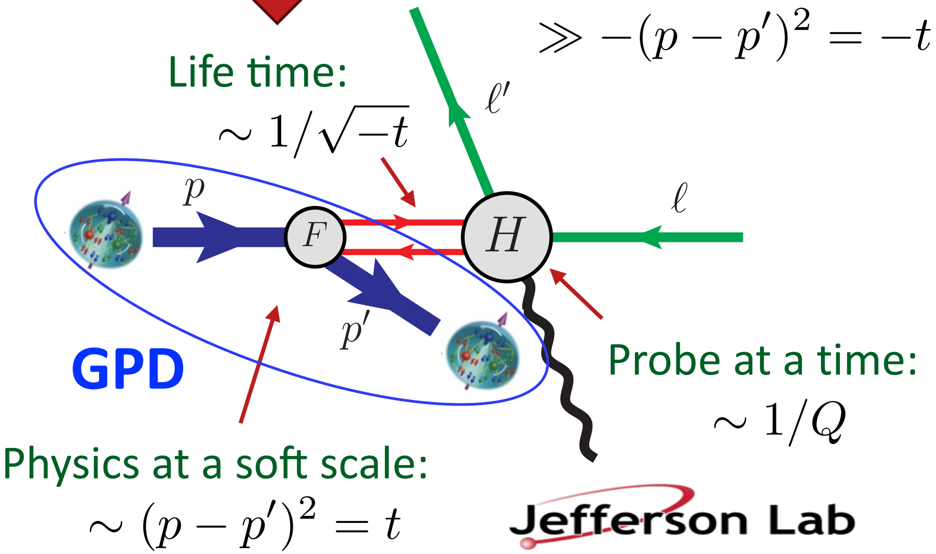




$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

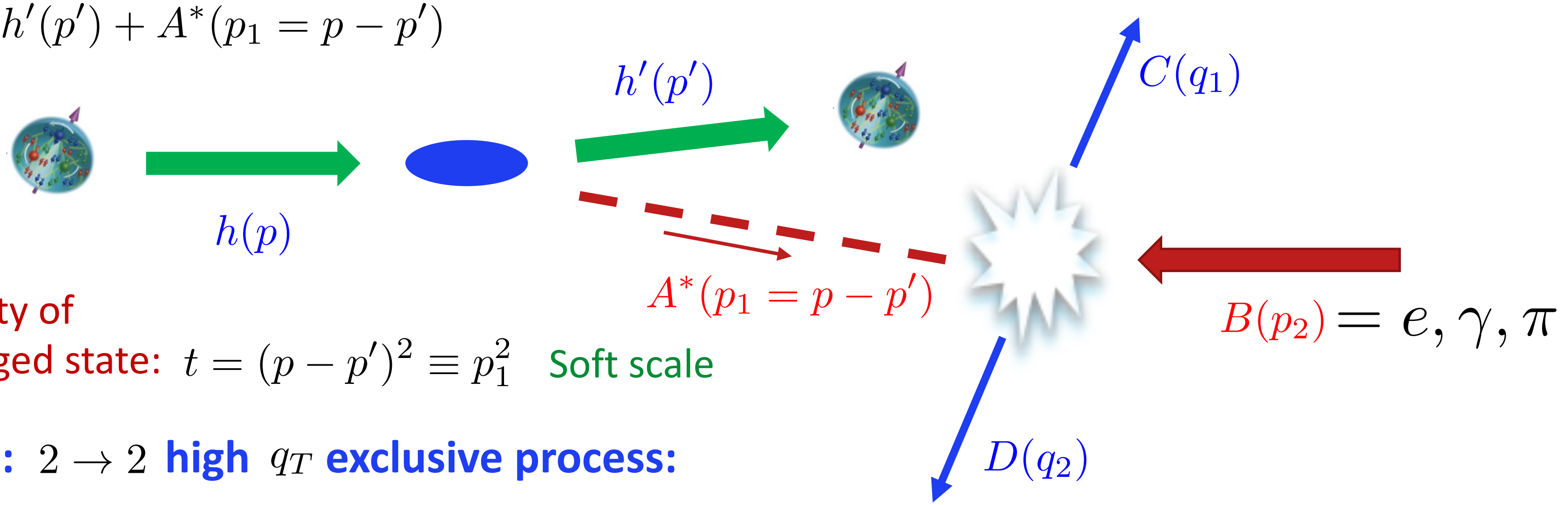
Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

❑ Separation of physics taken place at soft (t) and hard (Q) scales:

- **Single diffractive – keep the hadron intact:**

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$



Virtuality of exchanged state: $t = (p - p')^2 \equiv p_1^2$ Soft scale

- **Hard probe: $2 \to 2$ high q_T exclusive process:**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

Probing time: $\sim 1/|q_{1T}| \approx 1/|q_{2T}|$

➡ **Two- stage $2 \to 3$ single diffractive exclusive hard processes (SDHEP):**

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

A 2-scale 2-stage observable!

- **Necessary condition for QCD factorization:**

Lifetime of $A^*(p_1)$ is much longer than collision time of the probe! ➡ $|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$

Not necessarily sufficient!

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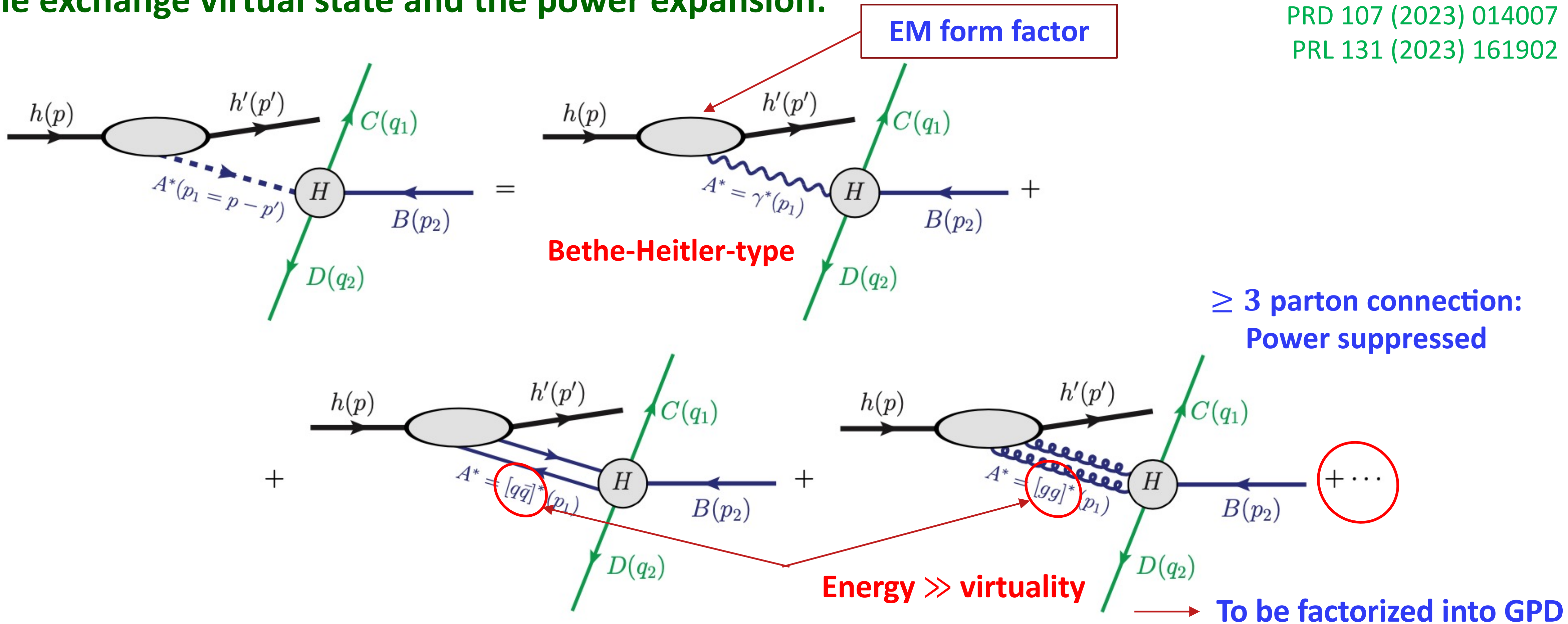
Single-Diffractive Hard Exclusive Processes (SDHEP)

❑ The exchange virtual state and the power expansion:

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902



The exchanged state $A^*(p\text{-}p')$ is a sum of all possible partonic states, n=1,2, … , allowed by

- Quantum numbers of *h(p) – h'(p')*
- Symmetry of producing non-vanishing *H*

Need to separate different contributions!

Proper angular modulations!

Generalized Parton Distributions (GPDs)

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

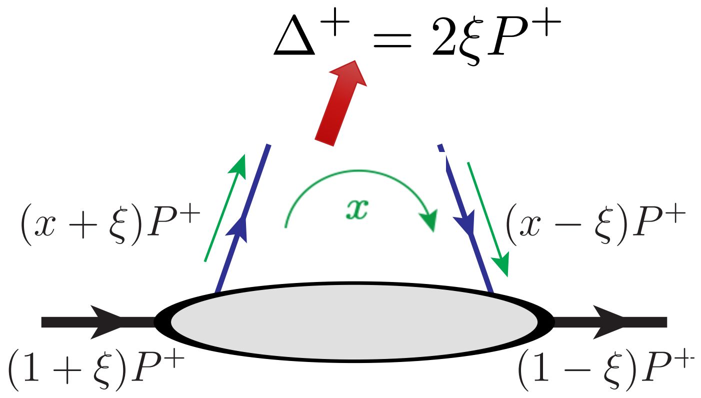
❑ Definition:

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m} u(p) \right].$$



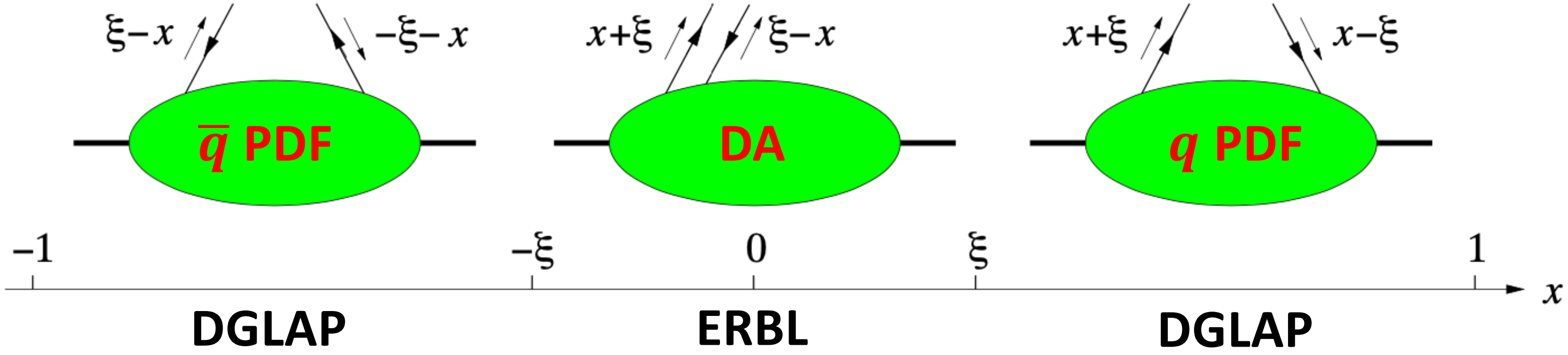
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

❑ Combine *PDF* and *Distribution Amplitude (DA):*

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



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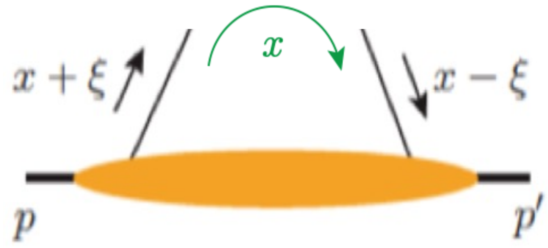
Properties of GPDs – Partonic

❑ **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$



Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

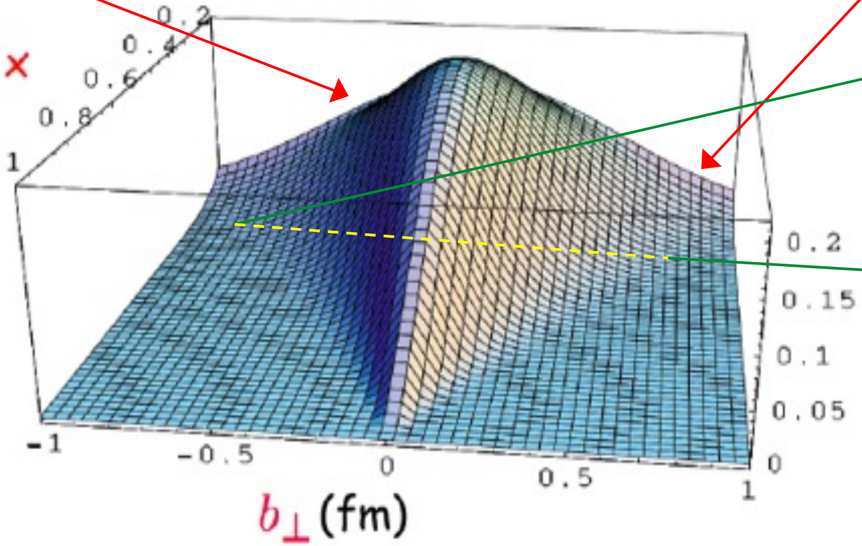
➡ **Quark density in** $\mathrm{d}x\, \mathrm{d}^2 \boldsymbol{b}_T$

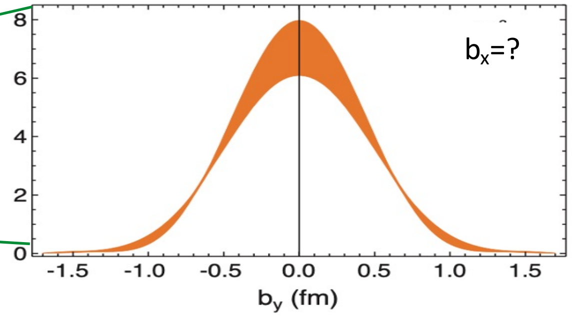
How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?

➡





Slice in (x,Q)

Modeled by M. Burkdart, PRD 2000

$$\langle q_\perp^N \rangle \equiv \int db_\perp b_\perp^N \, q(x, b_\perp, Q)$$

➡ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

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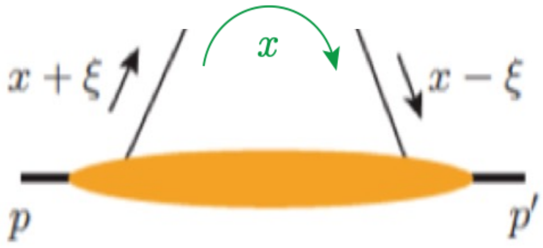
➡ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$



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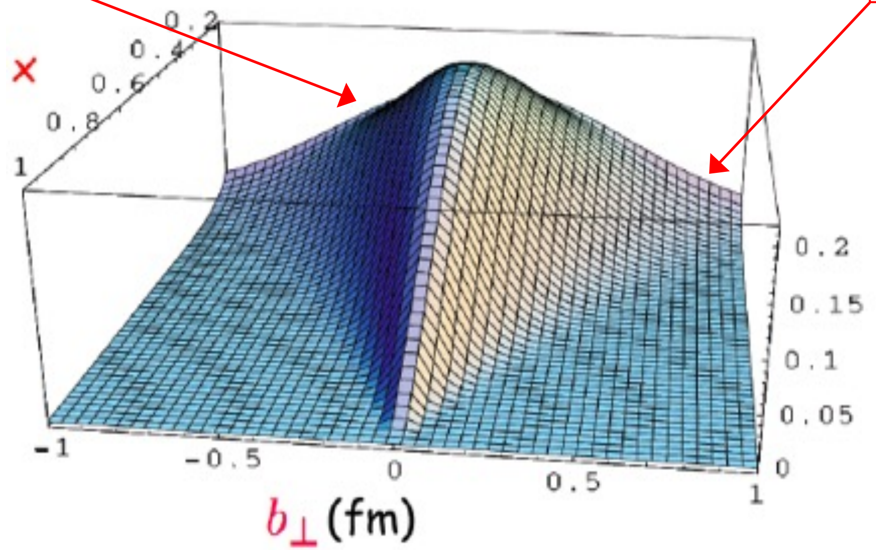

Modeled by M. Burkdart, PRD 2000

➡ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

- **Should** $r_q(x)$ **>** $r_g(x)$**, or vice versa?**
- **Could** $r_g(x)$ **saturates as** $x \to 0$
- **How do they compare with known radius (EM charge radius, mass radius, …), & why?**
- **How the image correlate to hadron spin, … ?**
- …

Properties of GPDs – Hadronic = Moments of GPDs

Ji, PRL78, 1997
V. D. Burkert, et al. RMP 95 (2023) 041002

❑ QCD energy-momentum tensor:

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta}\right)^2$$

❑ "Gravitational" form factors:

$$\langle p' | T_i^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{\left(A_i + \xi^2 D_i\right)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{\left(B_i - \xi^2 D_i\right)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

$C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)
Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

$$i = q, g$$

➡ **3D tomography**
Relation to GFFs
Angular Momentum ➡ ***Need x*-dependence of GPDs!**

Need to know the x-dependence of GPDs to construct the proper moments!

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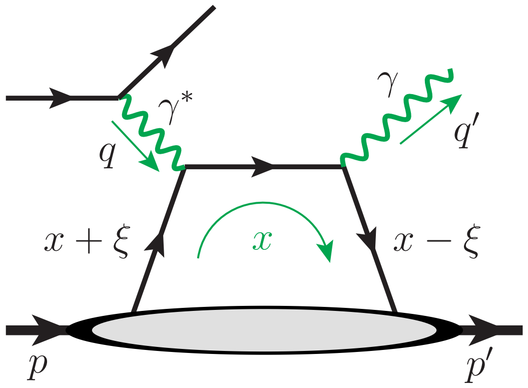
Exclusive Diffractive Processes for Extracting GPDs

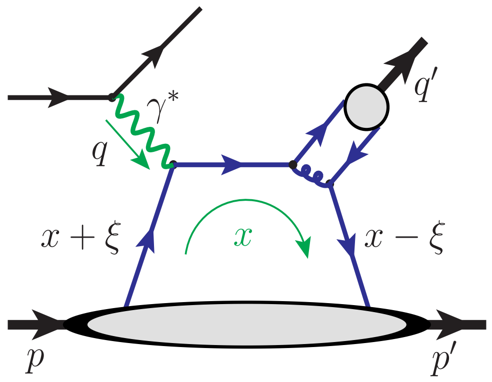
❑ Known exclusive processes for extracting GPDs:

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$



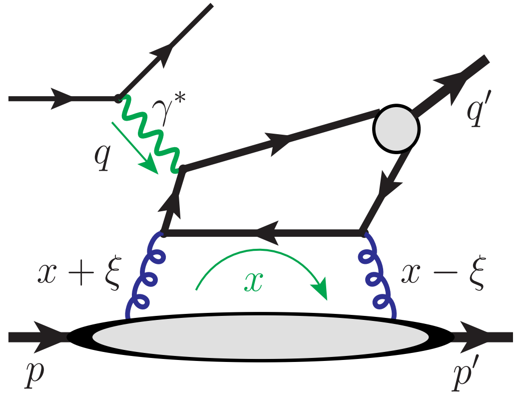
DVCS: Q² >> |t|

$B = e, C = e, D = \gamma$



DVMP

$B = e, C = e, D = \pi, \ldots$

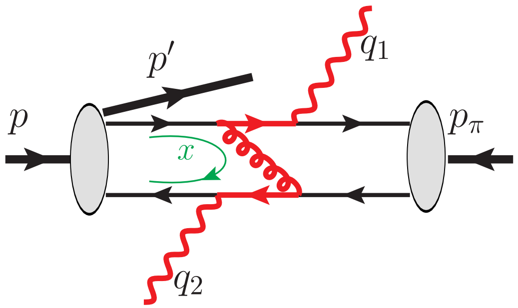


DVQP

$B = e, C = e, D = J/\psi$

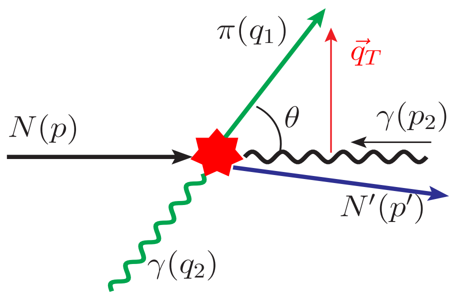
+ …

❑ New exclusive processes for extracting GPDs:



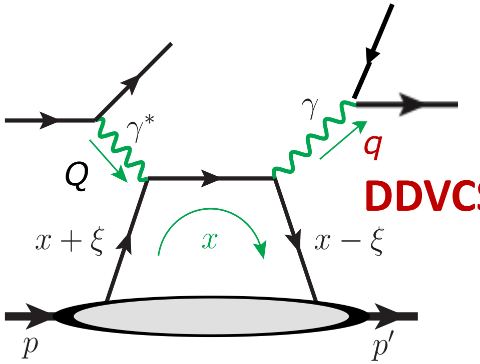
$B = \pi, C = \gamma, D = \gamma$

J-PARC, AMBER



$B = \gamma, C = \pi, D = \gamma$

JLab, EIC



DDVCS

$B = e, C = e, D = \gamma^*$

JLab

Better sensitivity on the x-dependence!

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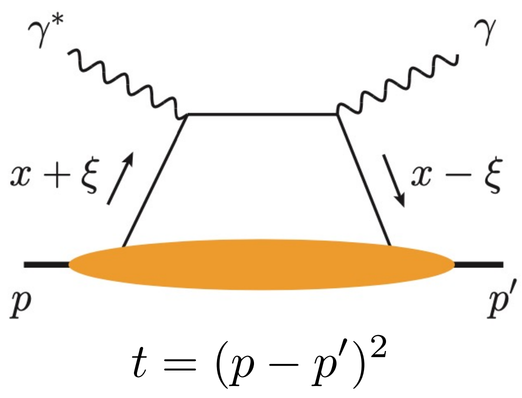
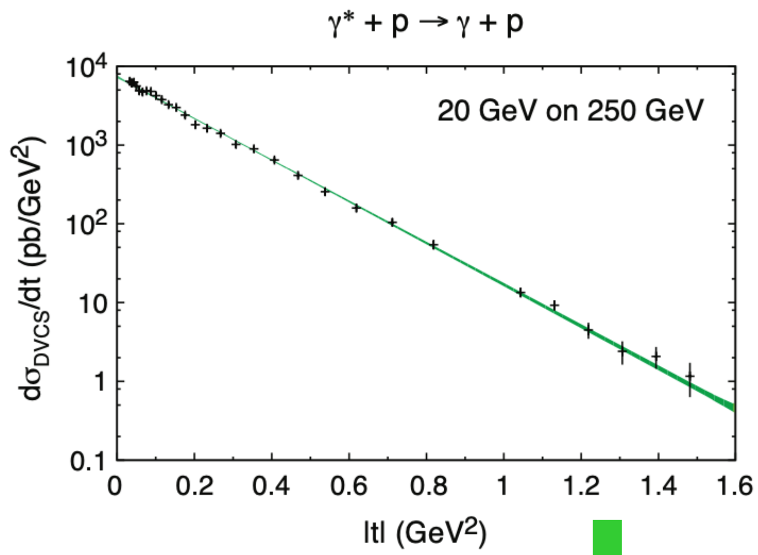
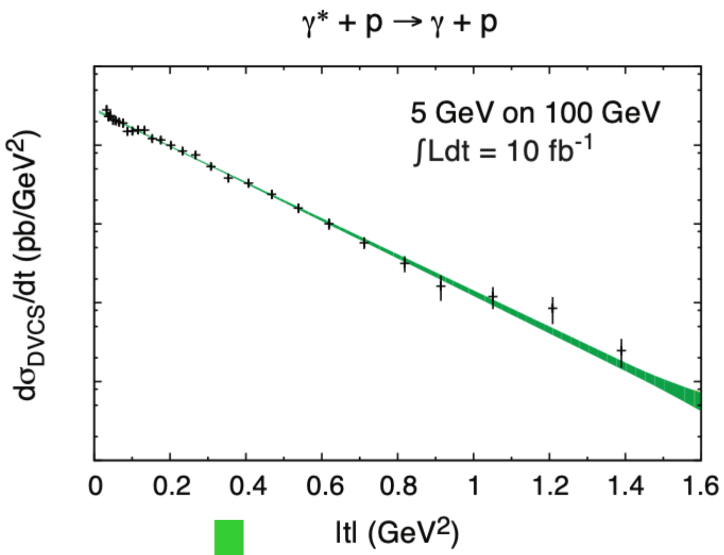
DVCS at the EIC (White Paper)

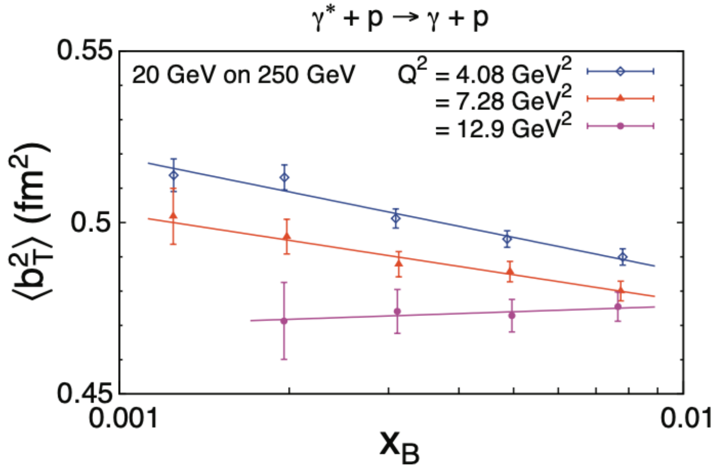
❑ Cross Sections:

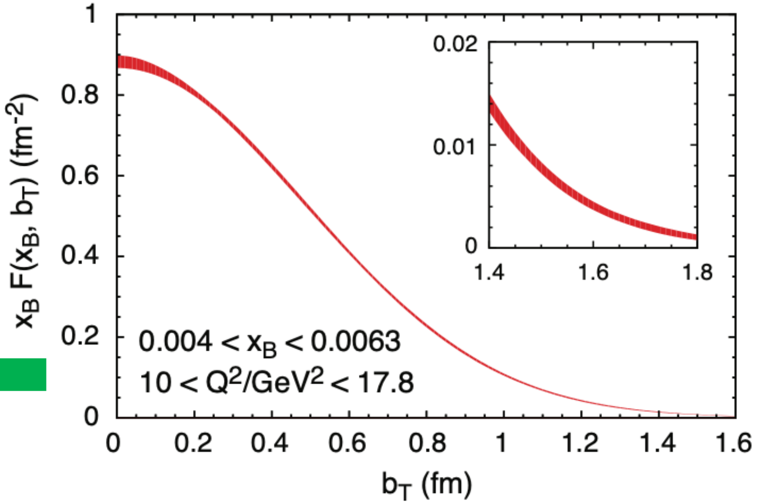


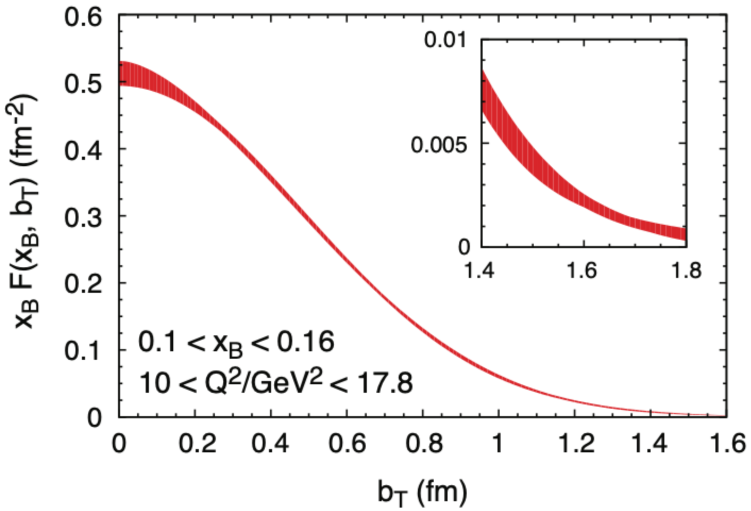




❑ Spatial distributions:







Effective "proton radius" in terms of quarks as a function of x_B

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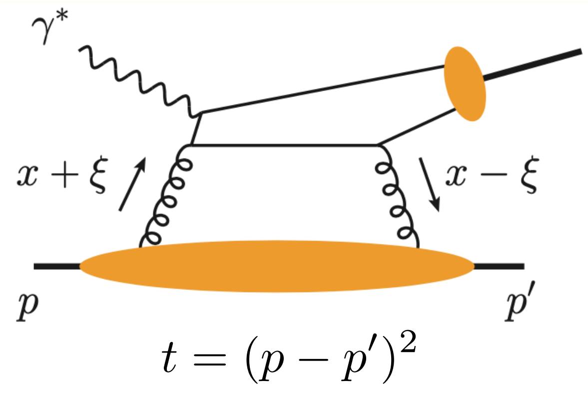
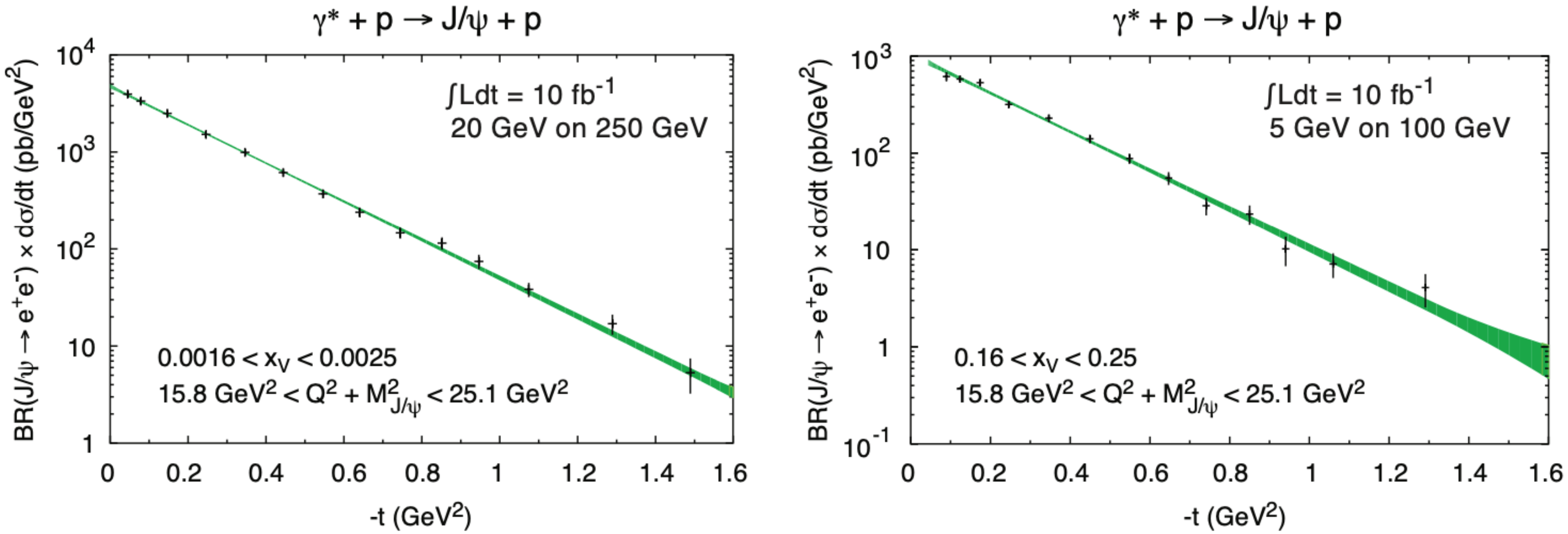
Imaging the Gluon at the EIC (White Paper)

❑ Exclusive vector meson production:





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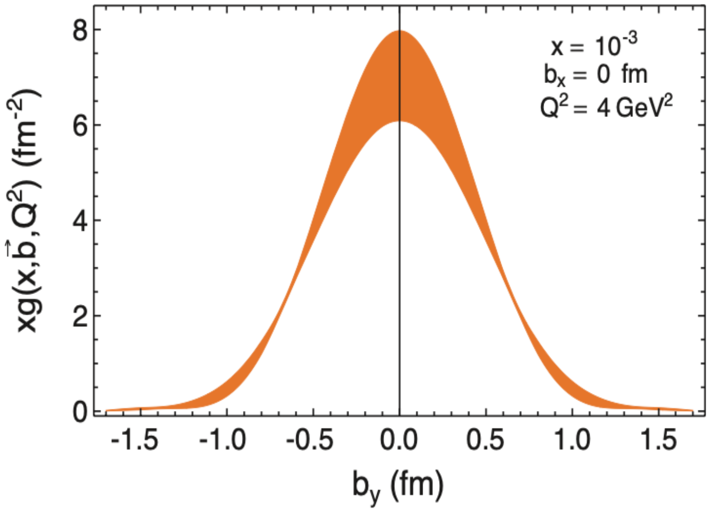

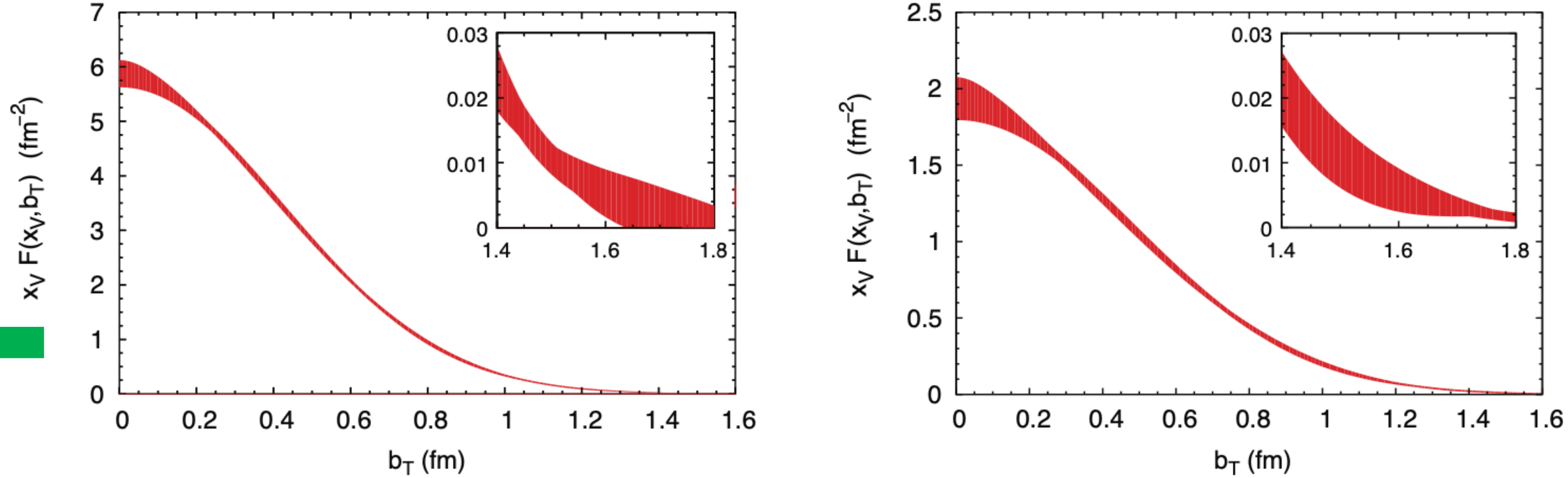

The b_T space density for gluons → **Effective "proton radius" in terms of gluons**

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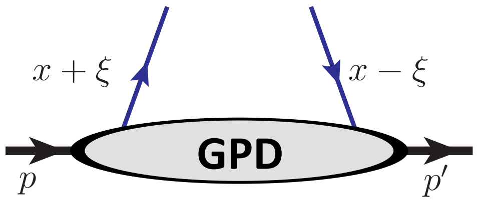
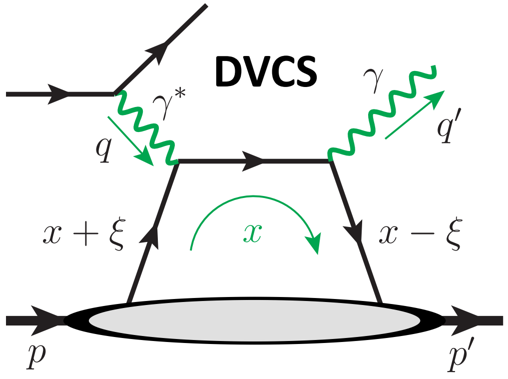
Why is the GPD's *x*-dependence so *difficult* to measure?

❑ Amplitude nature: exclusive processes



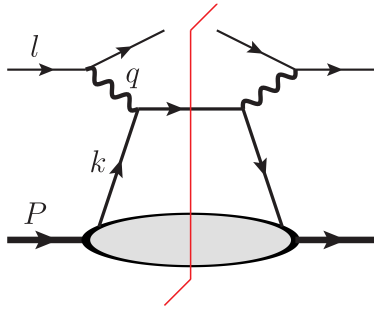
x ~ loop momentum

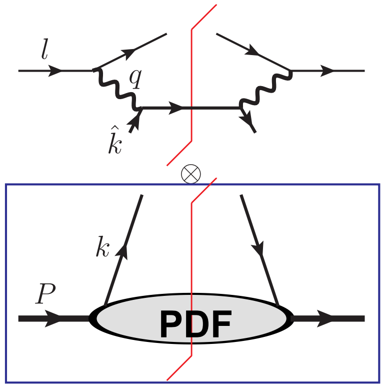


$$iM \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x=0$; $x=\pm\xi$

Compare with DIS





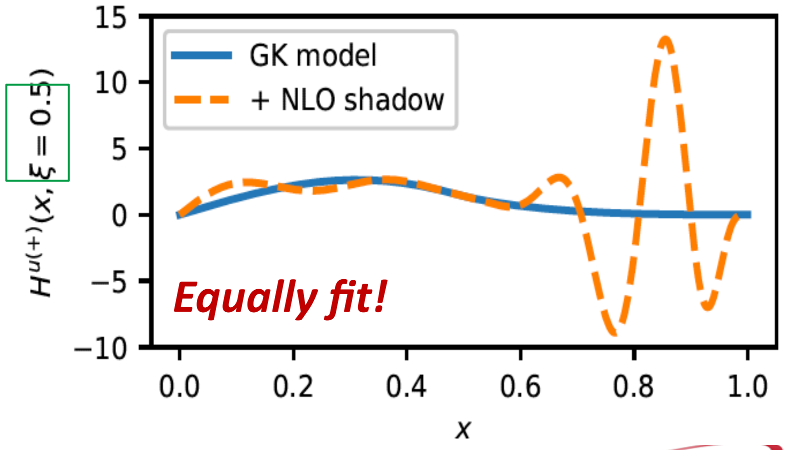
***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

❑ Sensitivity to *x*: comes from $C(x,\xi;Q/\mu)$

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

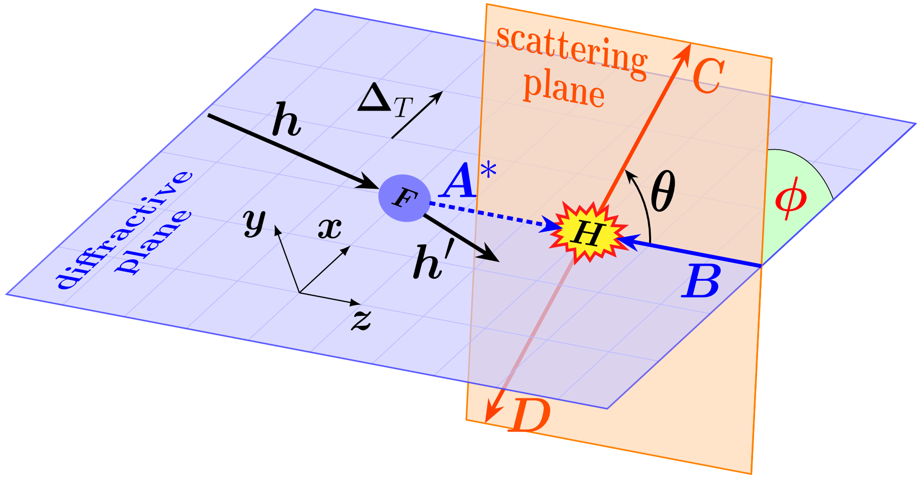
$$\Rightarrow\ iM \propto \int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$ **“moment”**



[Bertone et al. PRD `21]

Jefferson Lab

Where does the x-sensitivity come from?



- x-sensitivity $\Leftrightarrow$ 2 → 2 hard scattering:

Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⟵ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⟷ x
3. ϕ ⟵ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

- **Moment-type sensitivity:** $C(x;Q) = G(x)\cdot T(Q)$ ⟶ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q** **Scaling for F_G**

⟶ **Inversion problem:** ***shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

- **Enhanced sensitivity:** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟶ $d\sigma/dQ \sim |C(x;Q)\otimes_x F(x,\xi,t)|^2$

Moment-type Sensitivity: $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

PRD56 (1997) 5524; PRD58 (1998) 094018; PRD59 (1999) 074009

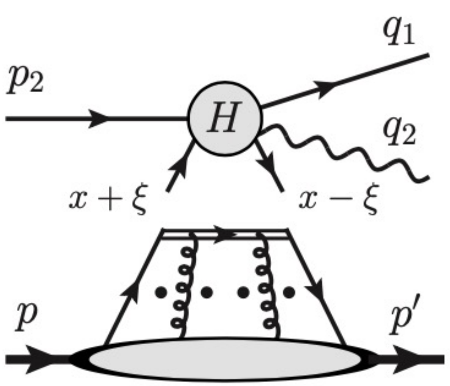
❑ DVCS:

$$h(p) = \text{Proton}(p),\ h'(p') = \text{Proton}(p'),\ B(p_2) = \text{electron}(p_2),\ C(q_1) = \text{electron}(q_1),\ D(q_2) = \text{photon}(q_2)$$

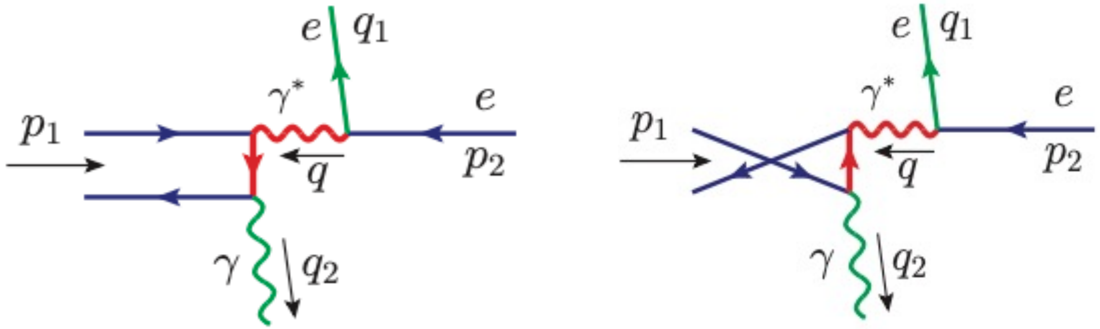
Factorization:

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$t = (p-p')^2$$



LO:



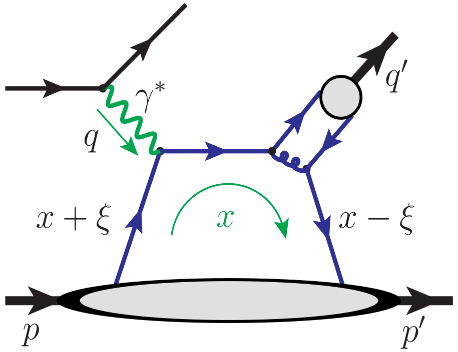
$$C^{(0)} \propto \frac{1}{x-\xi+i\varepsilon} - \frac{1}{x+\xi-i\varepsilon}$$

$$\mathcal{M}^{(2)}_{he\to h'e\gamma} = \sum_i \int_{-1}^{1} \mathrm{d}x\, F_i^h(x,\xi,t)\, C_{ie\to e\gamma}(x,\xi,q_T),$$

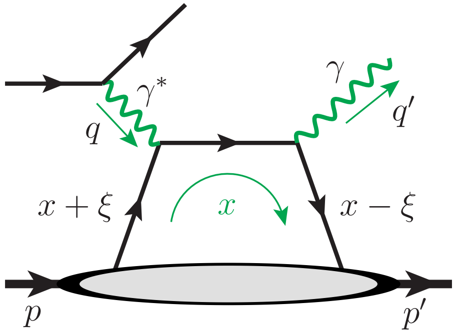
The x-integration is NOT sensitive to externally measured hard scale, q_T or Q^2!

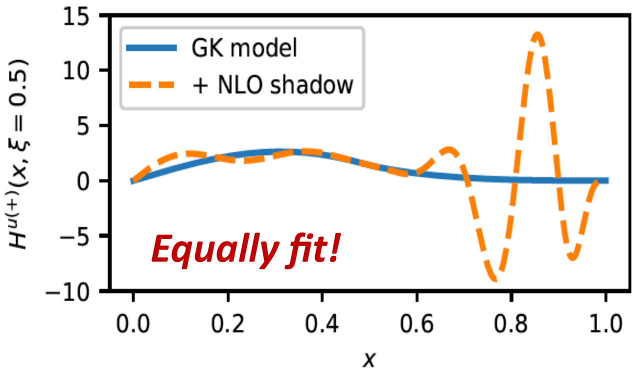
Need a very large range of Q^2, but, cross section is strongly suppressed!

❑ DVMP:



Similar to





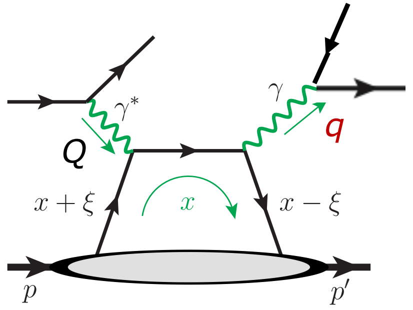
[Bertone et al. PRD `21]

What Kind of Process Could be Sensitive to the *x*-Dependence?

❑ **Create an entanglement between the internal *x* and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external *q* to sample different part of x.



- **Double DVCS (two scales):**

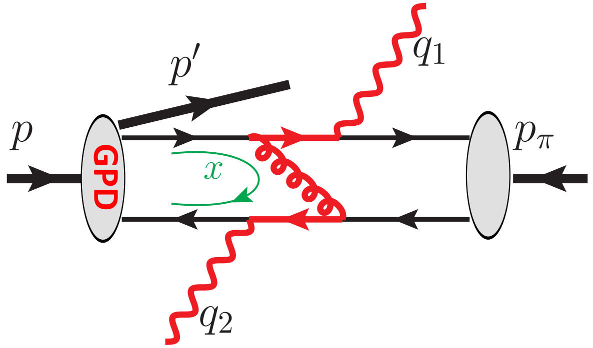
$$x_p(\xi,q) = \xi \left(\frac{1 - q^2/Q^2}{1 + q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Hard scale: $q_T \gg \Lambda_{\mathrm{QCD}}$ Soft scale: $t \sim \Lambda^2_{\mathrm{QCD}}$

Qiu & Yu
JHEP 08 (2022) 103



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu) \cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T)$$

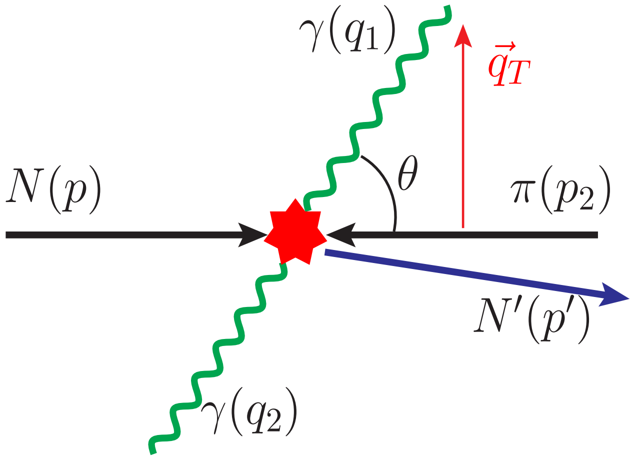
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\ \mathrm{d}\xi\ \mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

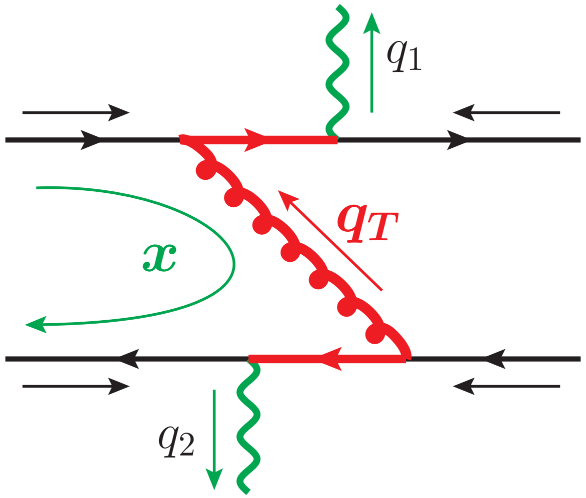
q_T distribution is "conjugate" to x distribution

$$x \leftrightarrow q_T$$

Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023





In addition to

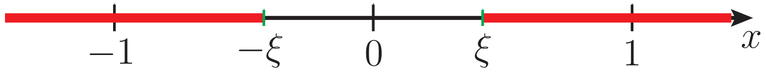
$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\xi+i\epsilon}$$

When two photons are radiated from the same charged line

$i\mathcal{M}$ **also contains**

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\rho(z;\theta)+i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2)-z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1-z+\tan^2(\theta/2)\,z}{1-z-\tan^2(\theta/2)\,z}\right] \in (-\infty,-\xi] \cup [\xi,\infty)$$



Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023

❑ Diphoton process: $N\pi \to N'\gamma\gamma$: (1) $p\pi^- \to n\gamma\gamma$; (2) $n\pi^+ \to p\gamma\gamma$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = 2\pi\left(\alpha_e\alpha_s\frac{C_F}{N_c}\right)^2\frac{1}{\xi^2 s^3}\cdot\Bigg[(1-\xi^2)\sum_{\alpha=\pm}\left(|\mathcal{M}_\alpha^{[\widetilde{H}]}|^2+|\widetilde{\mathcal{M}}_\alpha^{[H]}|^2\right)-\left(\xi^2+\frac{t}{4m^2}\right)\sum_{\alpha=\pm}|\widetilde{\mathcal{M}}_\alpha^{[E]}|^2 - \frac{\xi^2 t}{4m^2}\sum_{\alpha=\pm}|\mathcal{M}_\alpha^{[\widetilde{E}]}|^2 - 2\xi^2\sum_{\alpha=\pm}\mathrm{Re}\left(\widetilde{\mathcal{M}}_\alpha^{[H]}\widetilde{\mathcal{M}}_\alpha^{[E]*}+\mathcal{M}_\alpha^{[\widetilde{H}]}\mathcal{M}_\alpha^{[\widetilde{E}]*}\right)\Bigg]$$

Nucleon transition GPDs

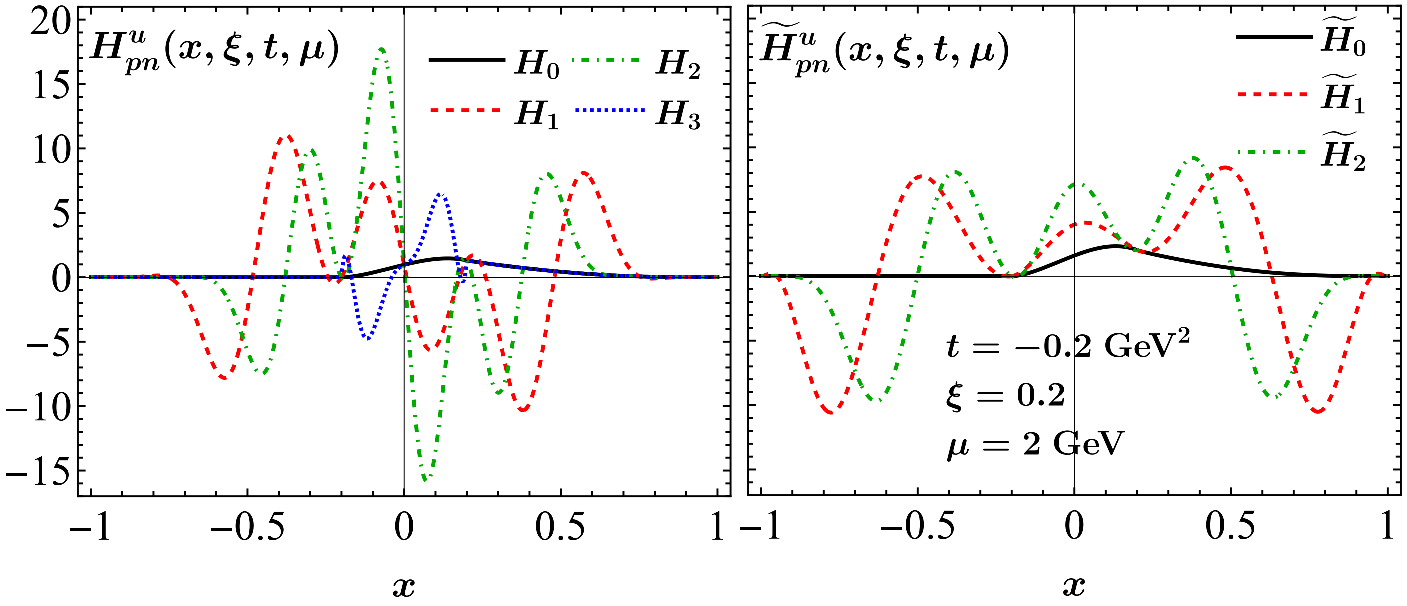
$$H_{pn}^u = H_p^u - H_p^d,\ \text{etc.}$$

GPD models = GK model + shadow GPDs

Goloskokov & Kroll, \`05, \`07, \`09

Bertone et al. \`21
Moffat et al. \`23

$$\int_{-1}^{1}\frac{dx\,S(x,\xi)}{x-\xi\pm i\epsilon}=0$$



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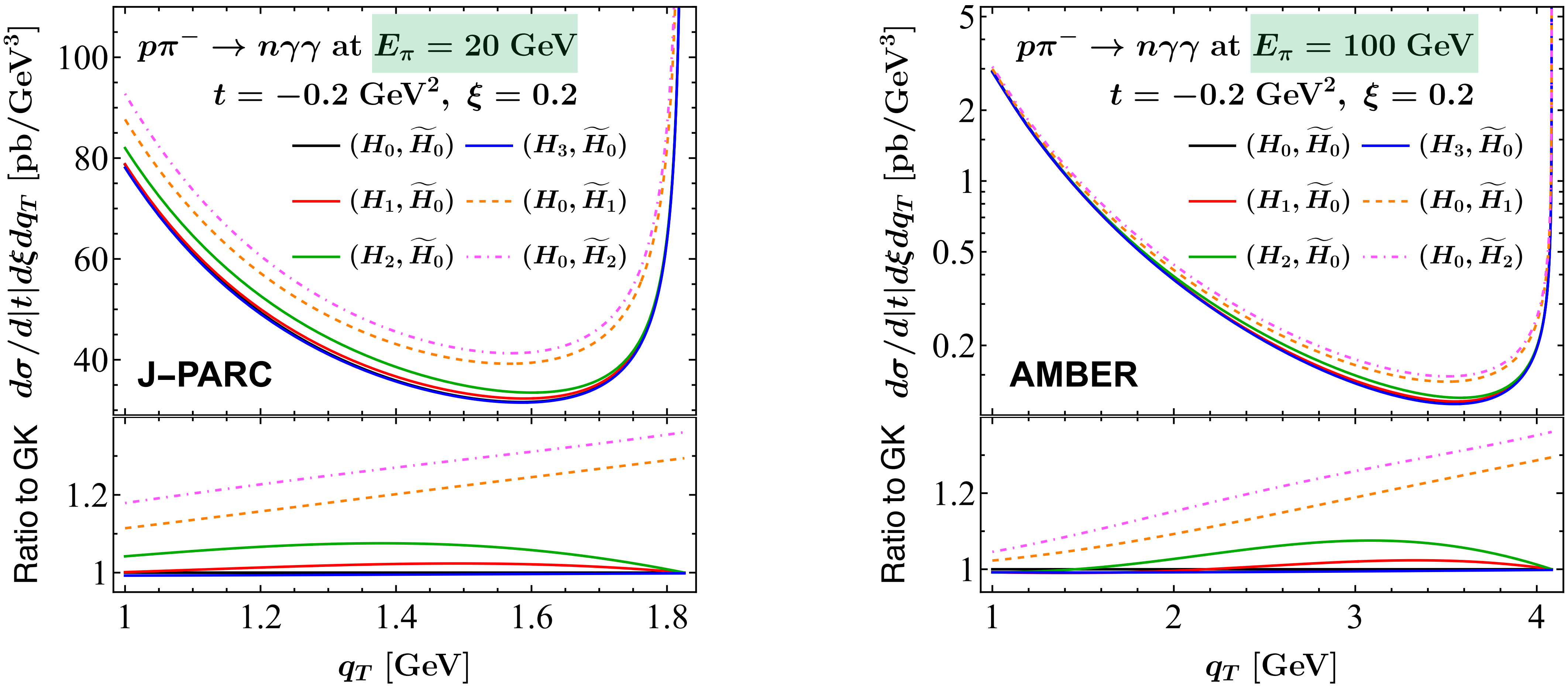
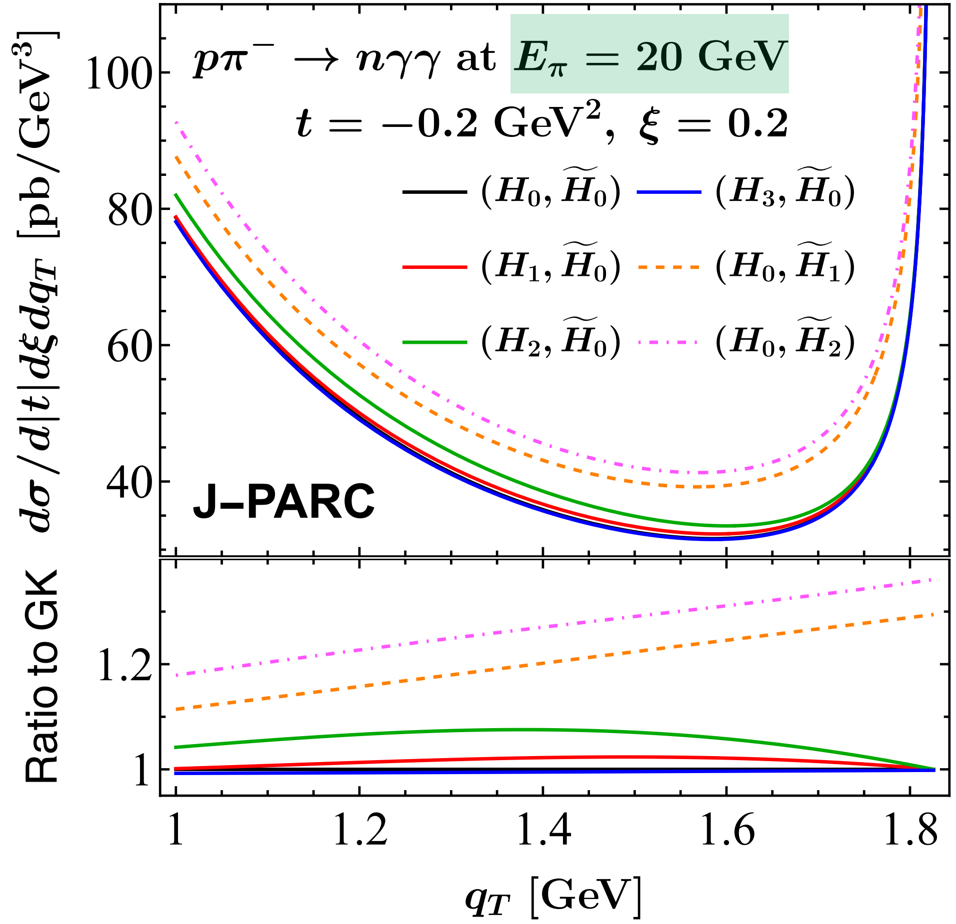
Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023



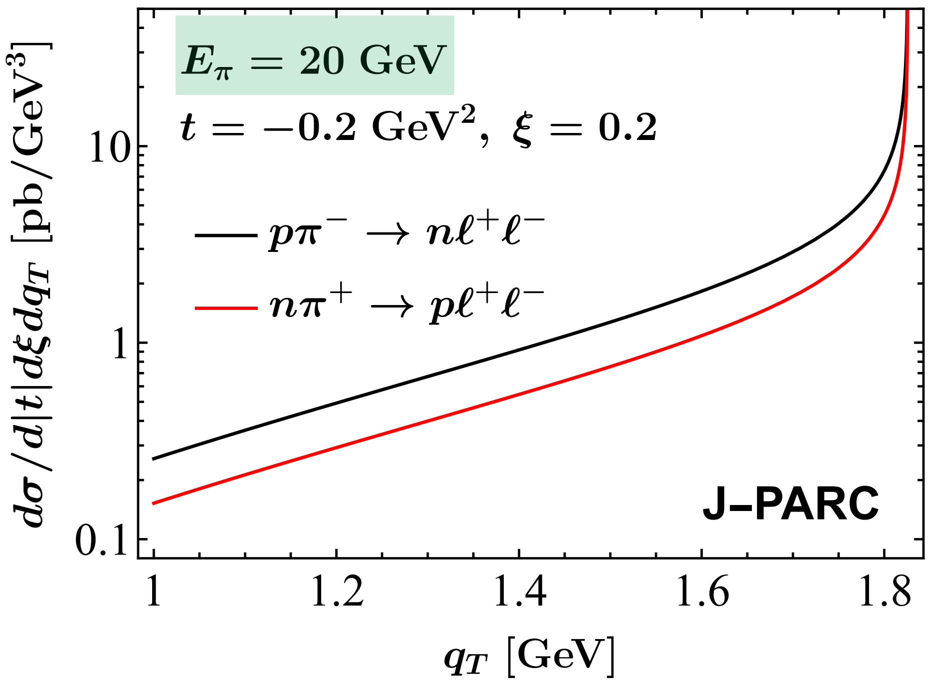
Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023



❑ Exclusive Drell-Yan dilepton production

$$N + \pi \to N' + \gamma^* [\to \ell^+ + \ell^-]$$



- Lower rate
- Blind to shadow GPDs

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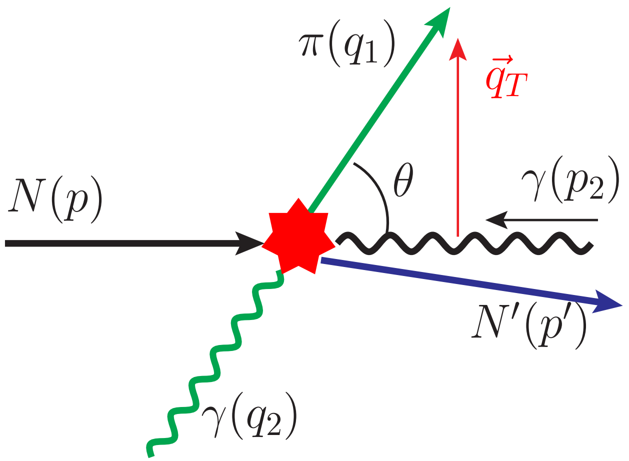
Enhanced x-Sensitivity: (2) γ-π Pair Photoproduction



G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRL 131 (2023), 161902

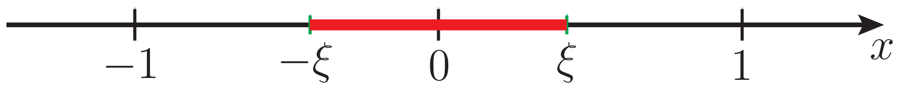
$i\mathcal{M}$ **also contains the special integral:**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

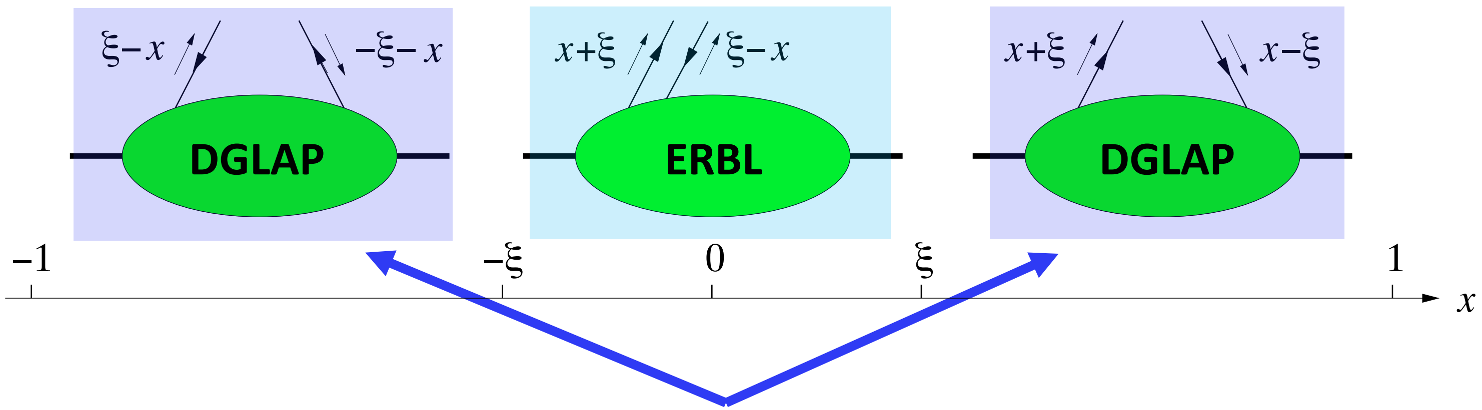
$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$

For DVCS/DVMP

$$\rho'(z,\theta) \to \xi$$



Complementary sensitivity:

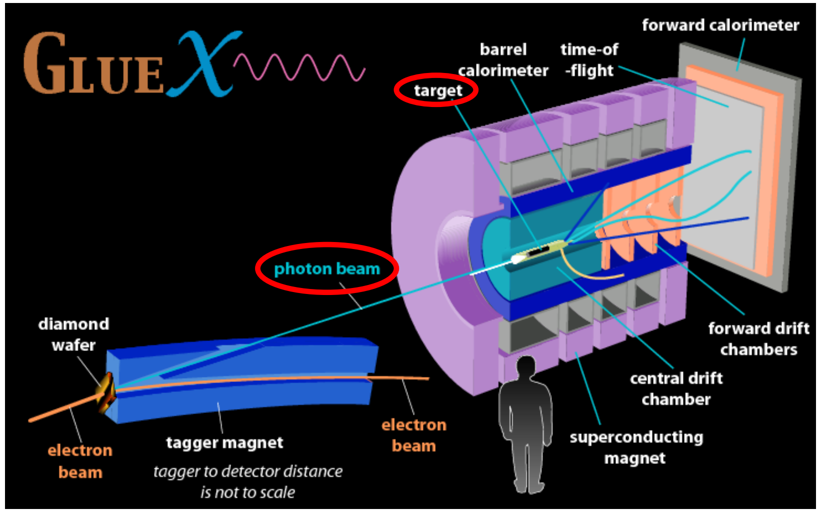


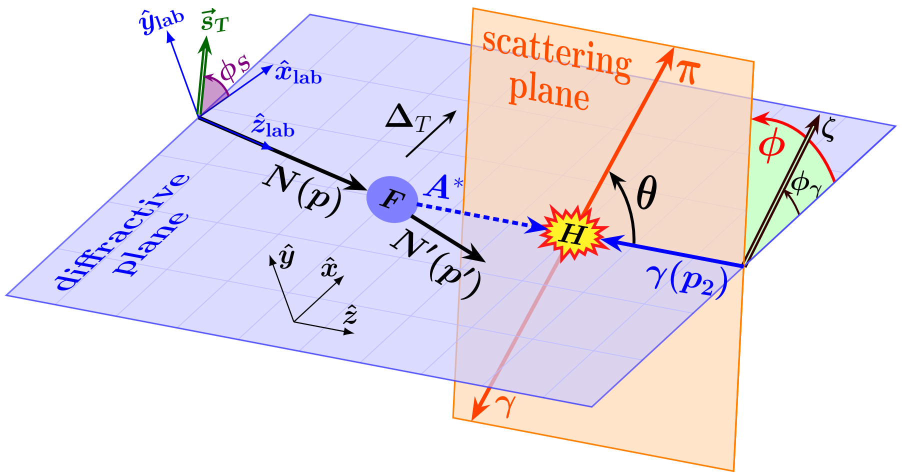
$N\,\pi \to N'\,\gamma\,\gamma$

Jefferson Lab

Enhanced x-Sensitivity: (2) γ-π Pair Photoproduction (at JLab Hall D)

Qiu & Yu, PRL 131 (2023), 161902





❑ Polarization asymmetries:

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\,\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma\,A_{LL} + \zeta\,A_{UT}\cos 2\left(\phi-\phi_\gamma\right) + \lambda_N\zeta\,A_{LT}\sin 2\left(\phi-\phi_\gamma\right)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\left(\alpha_e\alpha_s\right)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

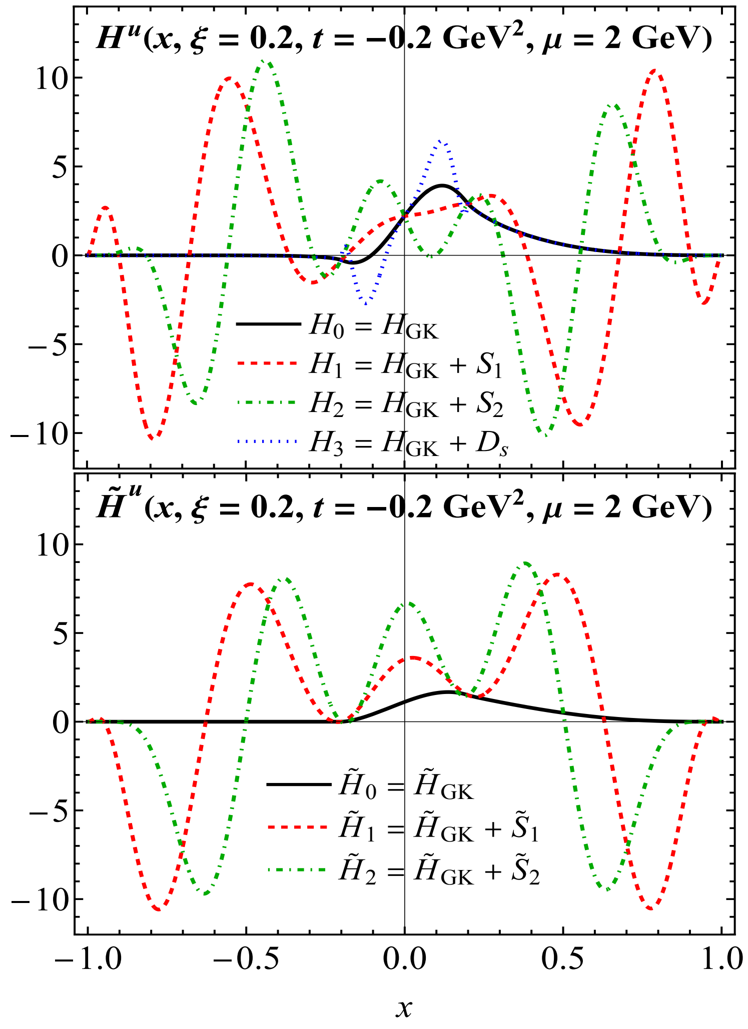
Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel

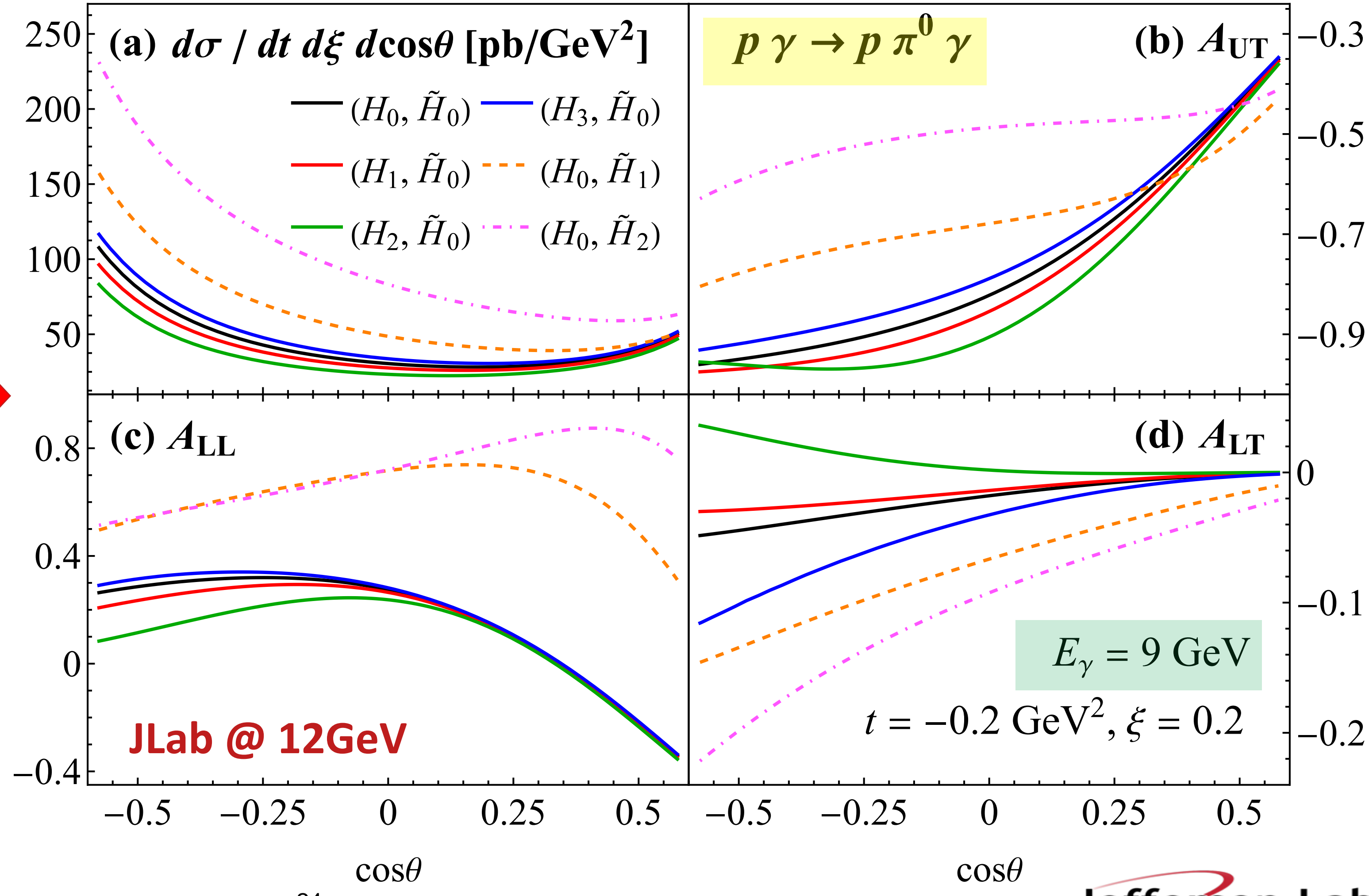
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$





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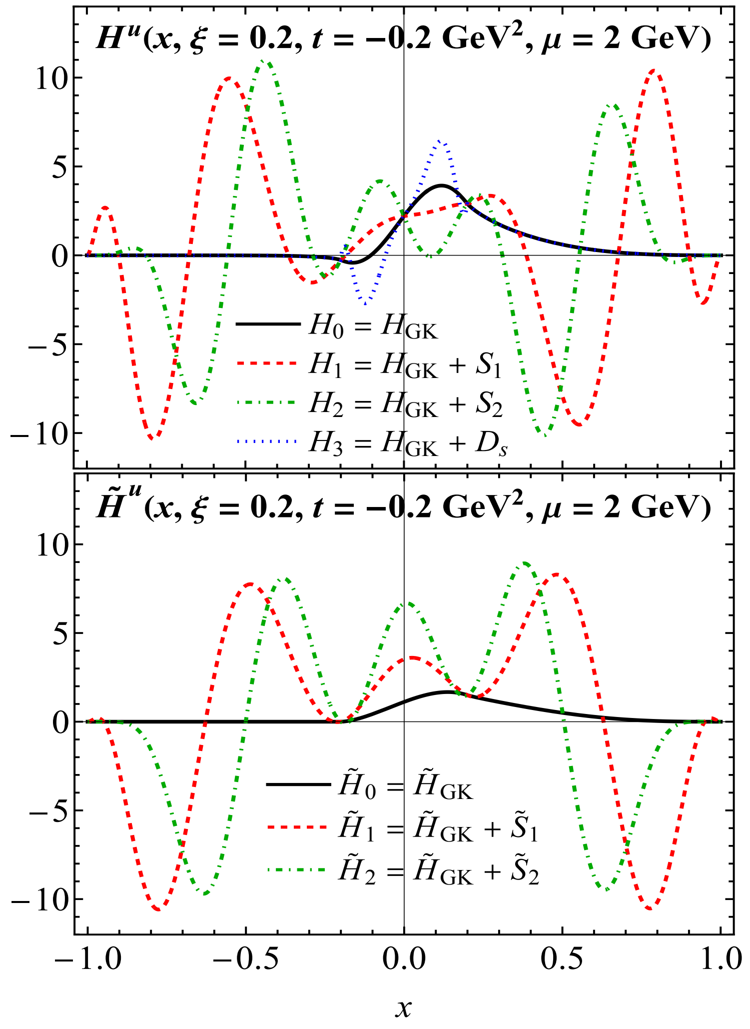
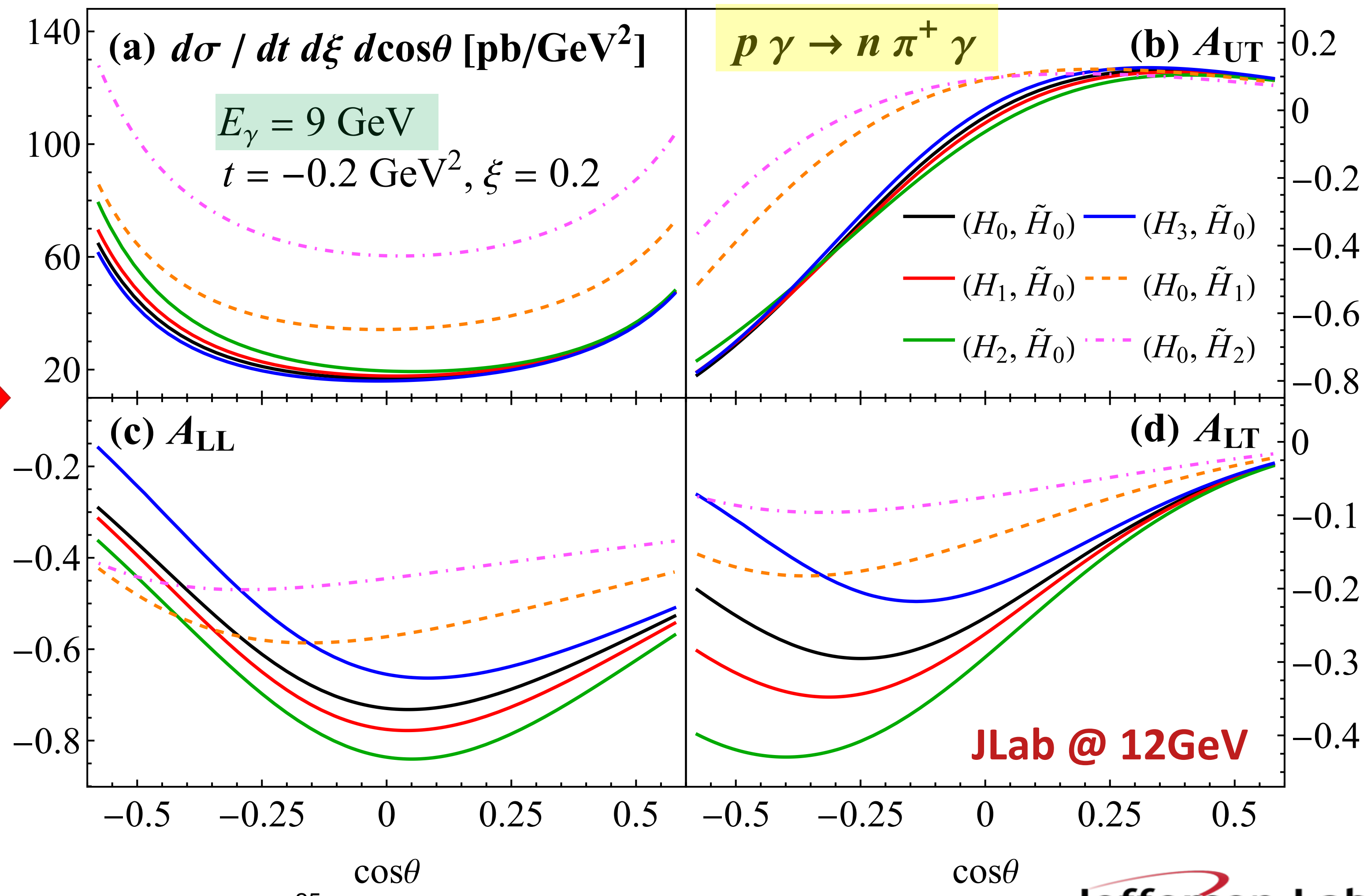
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$





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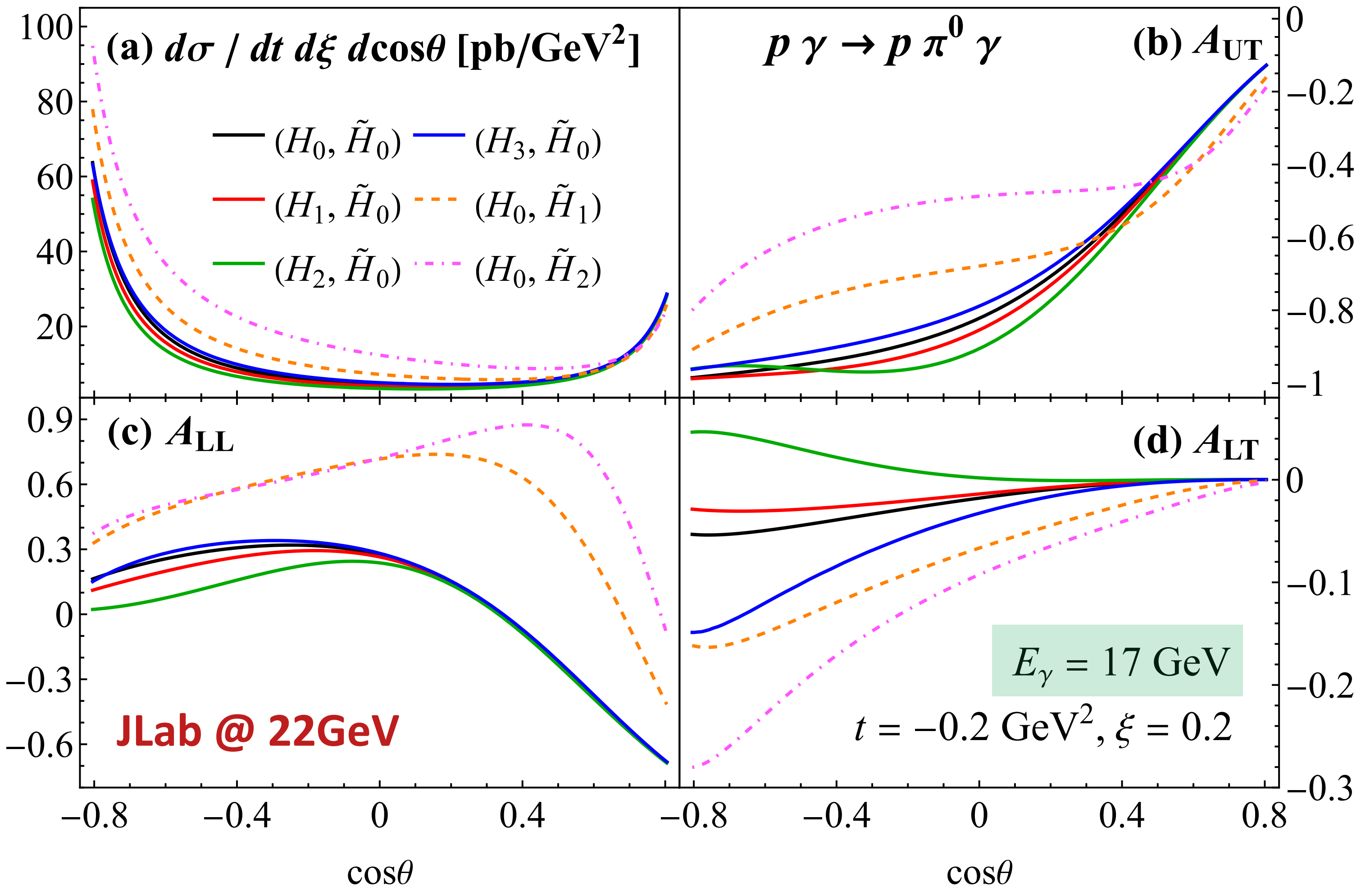
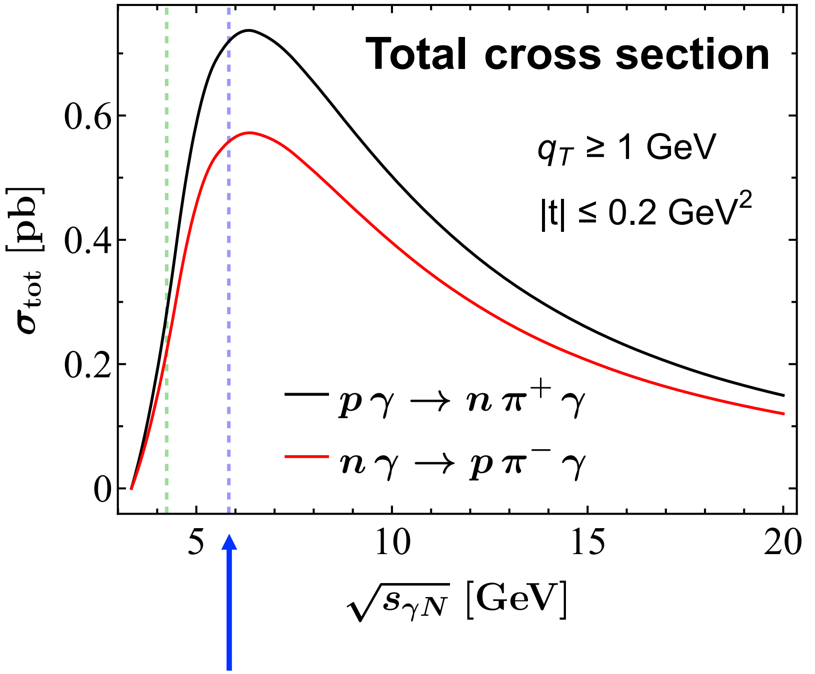
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at upgraded energy)

GPD models = GK model + shadow GPDs ⟵ $\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$

Goloskokov, Kroll, \`05, \`07, \`09
Bertone et al. \`21
Moffat et al. \`23
Qiu & Yu, \`23



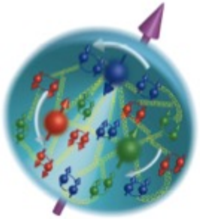


JLab @ 22GeV

A. Accardi et al. [arXiv:2306.09360]

How does the mass of the nucleon arise?

❑ Nucleon Mass – dominates the Mass of visible world!



Nucleon – a relativistic bound state of quarks and gluons

Mass is the **Energy** of the nucleon when it is at the **Rest**!

Mass = Rest Mass of quarks and gluons + **Energy of their motion**

❑ Higgs mechanism is NOT enough – mass without mass!

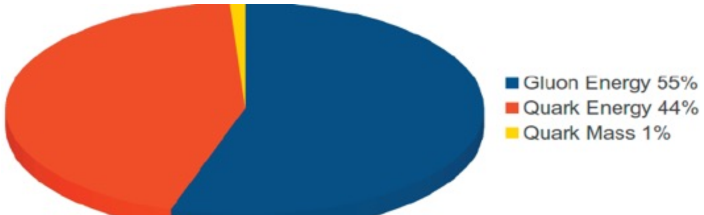


Fig. 2.1 NAS Report

Higgs mechanism is far from enough!!! ➡ **Energy of Confined Motion of quarks and gluons**

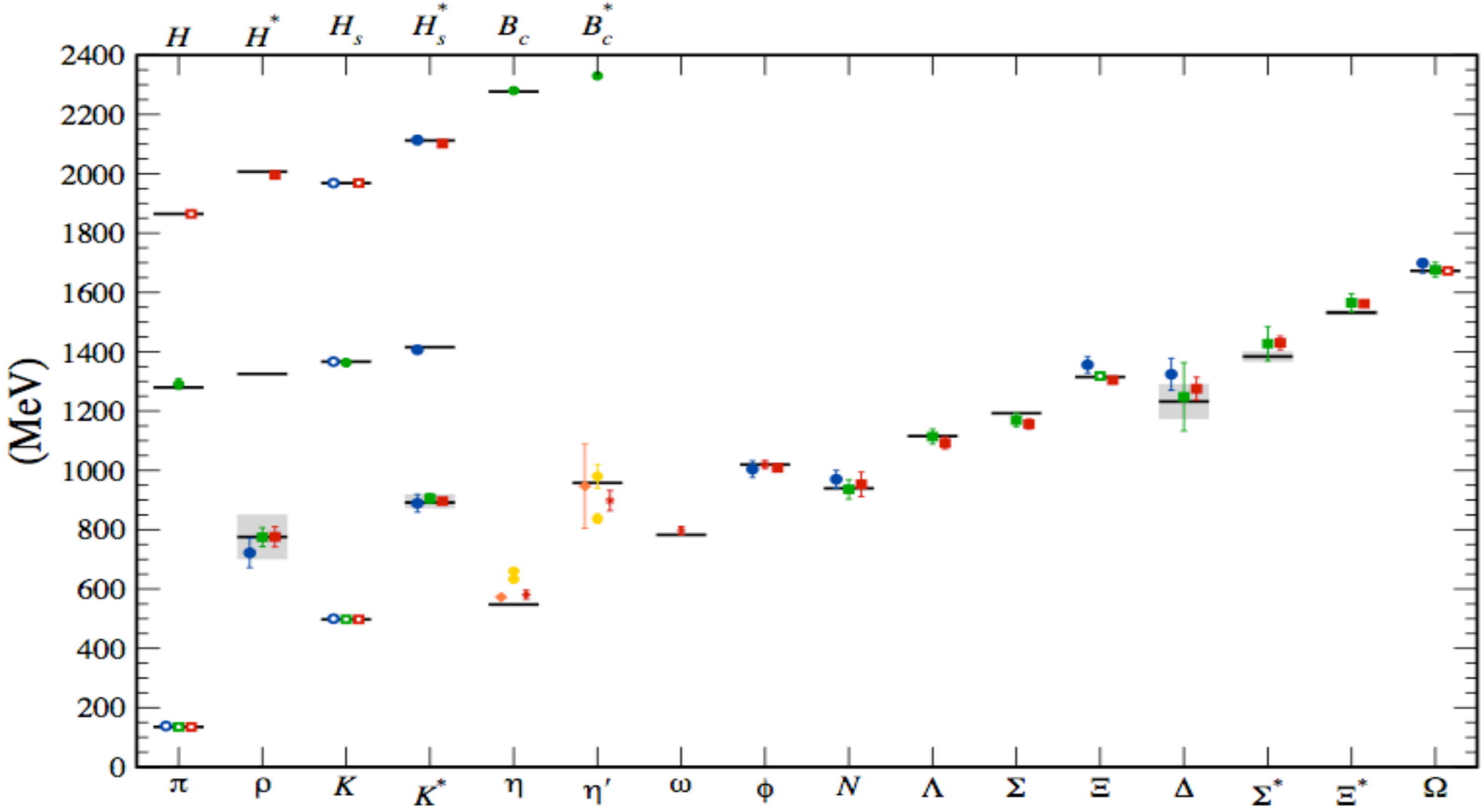
❑ Consistency check:

Bag model:

- **Kinetic energy of three quarks:** $K_q \sim 3/R$
- **Bag energy (bag constant B):** $T_b = \frac{4}{3}\pi R^3\, B$
- **Minimize** $K+T$ **:** $M_p \sim \frac{4}{R} \sim \frac{4}{0.84\text{fm}} \sim 938\ \text{MeV}$

Who ordered the hadron mass scale?

- Hadron mass from lattice QCD calculation:

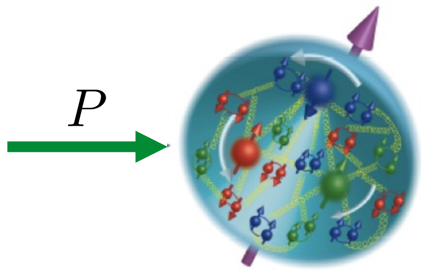


QCD is the right theory!

How to quantify and verify this, theoretically and experimentally?

Mass of Nucleon in QCD

❑ Decomposition of the trace of EMT:



Trace of the QCD energy-momentum tensor:

$$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} \underbrace{m_q (1+\gamma_m) \overline{\psi}_q \psi_q}_{\text{Chiral symmetry breaking}}$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$$

$$\langle P | T^{\alpha}_{\ \alpha}(0) | P \rangle = 2P^2 = 2M_n^2$$

➡ $$\langle T^{\alpha}_{\ \alpha} \rangle = \frac{\langle P | T^{\alpha}_{\ \alpha}(0) | P \rangle}{2P^0} = \frac{M_n^2}{P^0}$$

➡ $$M_n = \langle T^{\alpha}_{\ \alpha} \rangle |_{\text{at reest}}$$

Without separating the quark from gluon contribution to EMT

In the nucleon's rest frame, $$\underbrace{\langle \textstyle\int d^3r\, T^{\mu}_{\ \mu} \rangle}_{=M} = \underbrace{\langle \textstyle\int d^3r\, T^{00} \rangle}_{=M} - \sum_i \underbrace{\langle \textstyle\int d^3r\, T^{ii} \rangle}_{=0}$$

Nucleon mass: ***Gluon quantum effect + Chiral symmetry breaking!***

The sigma-term can be calculated in LQCD, Need the trace anomaly to test the sum rule!

Mass of Nucleon in QCD

C. Lorcé and et al, JHEP11 2021

- **Decompositions of $\sum_{f=q,g} T_f^{00}$:**

$$M = [\langle \int d^3r\, \overline{\psi}\gamma^0 iD^0\psi\rangle - \langle \int d^3r\, \overline{\psi}m\psi\rangle] + \langle \int d^3r\, \overline{\psi}m\psi\rangle + \langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle$$

Quark kinetic and potential energy	Quark rest mass energy	Gluon total energy

- **Ji's decomposition:**

X. Ji, PRL 1995

$$T_a^{00} = \underbrace{\bar{T}_a^{00}}_{= \frac{3}{4}T_a^{00} + \frac{1}{4}\sum_i T_a^{ii}} + \underbrace{\hat{T}_a^{00}}_{= \frac{1}{4}T_a^{00} - \frac{1}{4}\sum_i T_a^{ii}} \qquad a = q, g$$

$$M_n = \sum_{f=q,g} \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\Bigg|_{\text{cm}} = M_q + M_g + M_m + M_a$$

- Quark Energy $\langle \bar{T}_q^{00}\rangle$ → M_q
- Gluon Energy $\langle \bar{T}_g^{00}\rangle$ → M_g
- Quark Mass $\langle \hat{T}_q^{00}\rangle$ → M_m
- Trace Anomaly $\langle \hat{T}_g^{00}\rangle$ → M_a
- Relativistic motion → $M_q + M_g$
- χ Symmetry Breaking → M_m
- Quantum fluctuation → M_a

Different interpretation!

Mass of Nucleon in QCD

- **Ji's interpretation:**

Note: $\langle x\rangle_f$ and σ_q are calculable in lattice QCD

Quark Energy $\langle T_q^{00}\rangle$: $$M_q = \frac{3}{4}\left(M\sum_q \langle x\rangle_q - \sum_q \sigma_q\right)$$

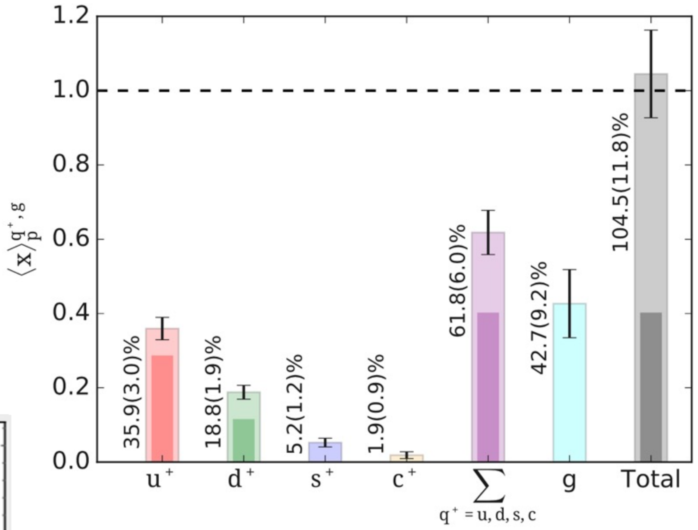
Gluon Energy $\langle T_g^{00}\rangle$: $$M_g = \frac{3}{4}M\langle x\rangle_g$$

Quark Mass $\langle \widehat{T}_q^{00}\rangle$: $$M_m = \sum_q \sigma_q$$

Trace Anomaly $\langle \widehat{T}_g^{00}\rangle$: $$M_a = \frac{\gamma_m}{4}\sum_q \sigma_q - \frac{\beta(g)}{4g}(E^2+B^2)$$

Parton momentum fraction:

$$\langle x\rangle_f = \int_0^1 dx\, x f(x,\mu^2)$$

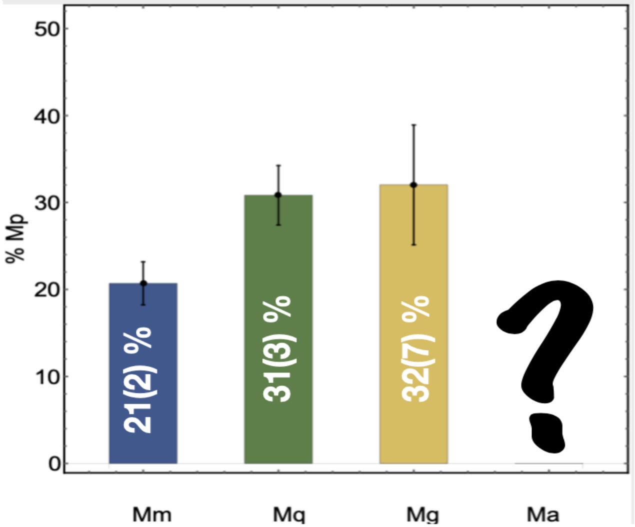


- **LQCD calculation:**

Quark sigma-term:

$$\sigma_q = \frac{\langle P|\overline{\psi}_q(0) m_q \psi_q(0)|P\rangle}{2P^0}$$

	$u+d$	s	c
σ [MeV]	41.6(3.8)	45.6(6.2)	107(22)



Access the trace anomaly Indirectly?

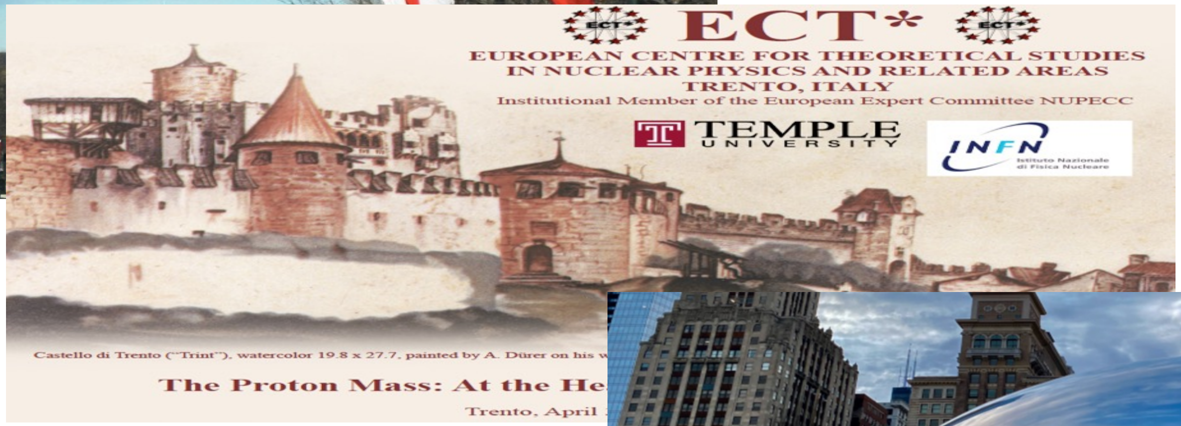
$$M_a = \frac{M}{4} - \sum_q \frac{\sigma_q}{4}$$

Or by experiment?

Jefferson Lab

The Proton Mass – What is the next?

- **Three-pronged approach to explore the origin of hadron mass**
  - **Lattice QCD**
  - **Mass decomposition – roles of the constituents**
  - **Model calculation – approximated analytical approach**



INT workshop (INT-20-77W):

Origin of the Visible Universe:

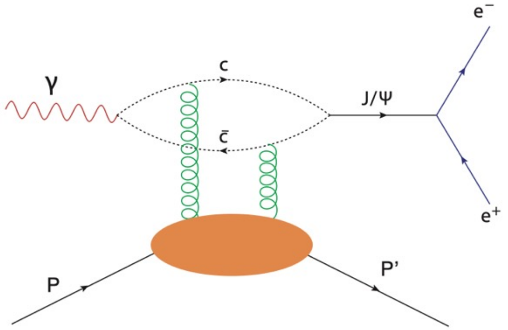
Unraveling the Proton Mass

June 13-17, 2022,

I. Cloet, Z.-E. Meziani, B. Pasquini

Actively thinking where to hold the next one?

- **Finding the trace anomaly:**

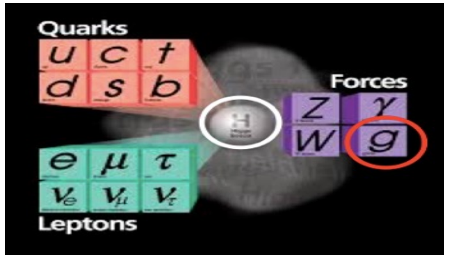


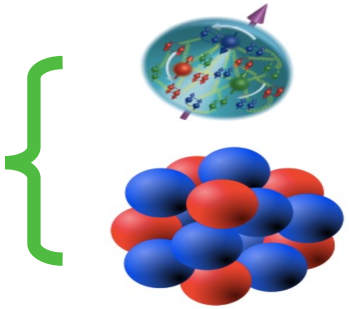
Two gluons may not be factorized into

$$F^{\mu\nu,a}F^{a}_{\mu\nu}$$

Emergent Properties of Dense Systems of Gluons?

- ❑ **Understanding the Glue that binds us all:**

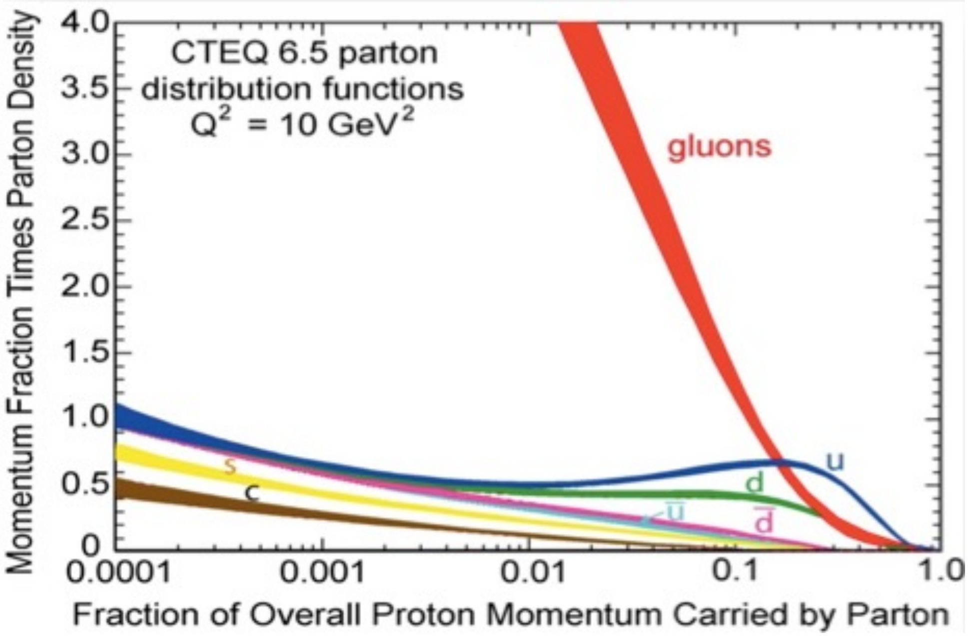




- ❑ **Gluons are weird particles!**
  - Massless, yet, responsible for a lot of visible mass
  - Carry color charge, unlike photon, responsible for color confinement, but, also for asymptotic freedom, as well as the abundance of glue!

Without gluons, there would be

NO nucleons, NO atomic nuclei, … NO visible world!



  - *What are the emergent properties of dense systems of gluons?*
  - *What does a nucleus look like if we only see quarks and gluons?*
  - *What is the coherent length of color force? …*

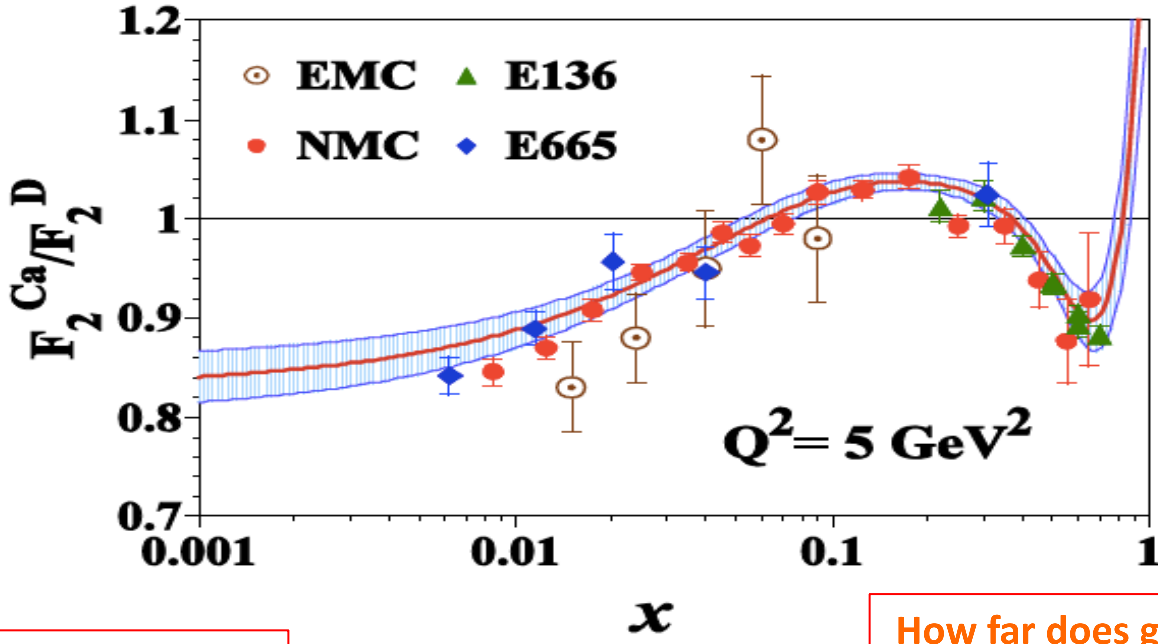
Nuclear Landscape as "seen" by a Hard Probe?

❑ EMC discovery – EMC effect:

Nuclear landscape

$\neq$ Superposition of nucleon landscape

- *What is the origin of nuclear force?*

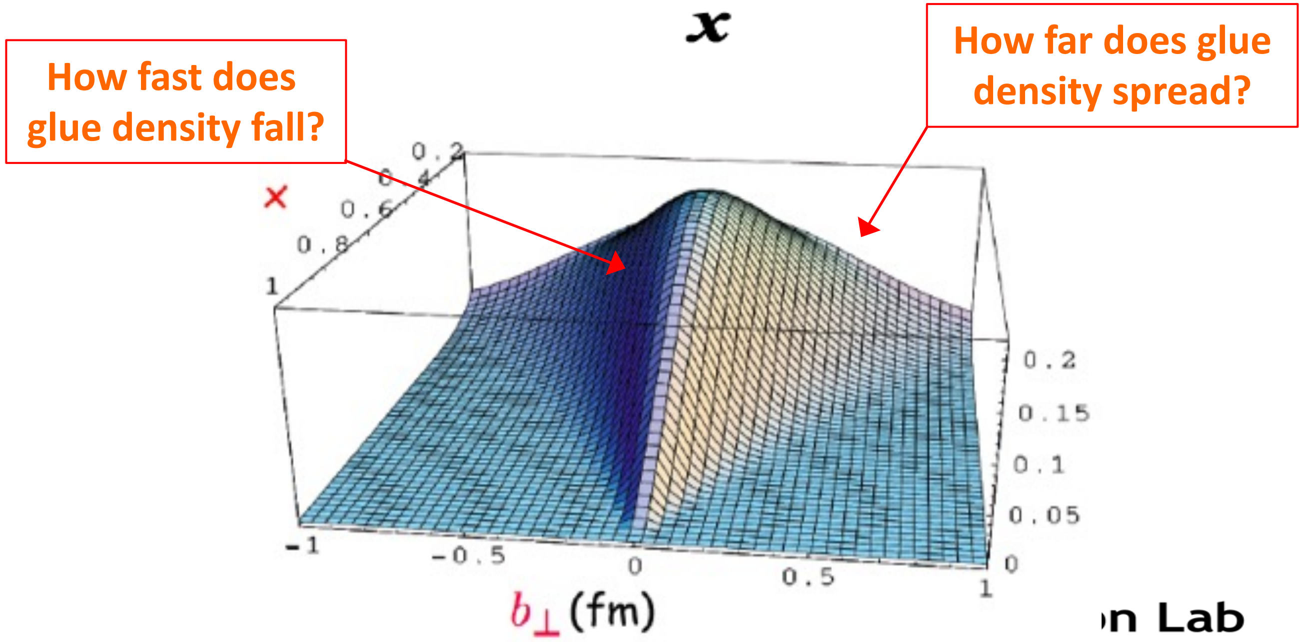


❑ Imaging the glue – only possible at EIC

- ✧ Gluon GPDs
- ✧ Discover the proton radius of gluon spatial distribution?

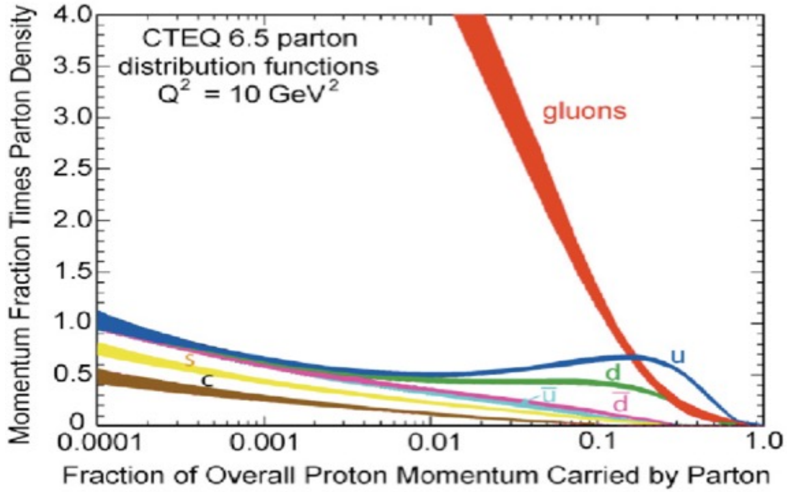
➡ **Proton radii from quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$**

Will the runaway gluon numbers at small-x lead to gluon saturation – Color Glass Condensate and an emergent mass scale ?



Gluon Saturation – Color Glass Condensate

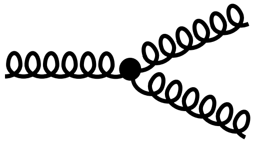
❑ Run away gluon density at small-x?



What causes the low-x rise?

gluon radiation

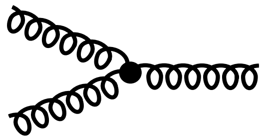
– non-linear gluon interaction



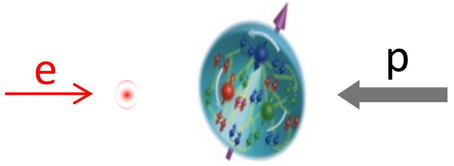
What could tame the low-x rise?

gluon recombination

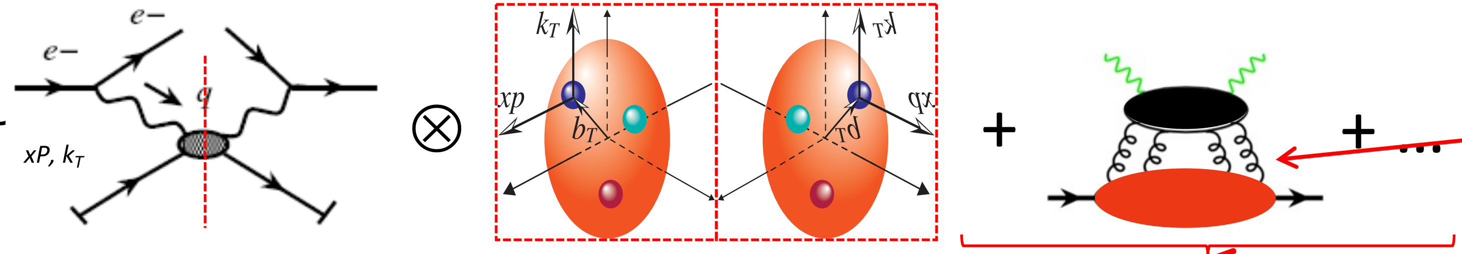
– non-linear gluon interaction



❑ Color entanglement enhanced at small-x:



$$\sigma^{\rm DIS}_{\rm tot} \sim$$



Color entangled or correlated between two active partons

$$\mathcal{O}\left(\frac{1}{QR}\right) \rightarrow \mathcal{O}\left(\frac{1}{QR} xg(x, Q_s)\right)$$

$$= \sum_f \hat{C}_f \otimes \Phi_f + \mathcal{O}\left(Q_s^2/Q^2\right) + \mathcal{O}\left(Q_s^4/Q^4\right) + \ldots$$

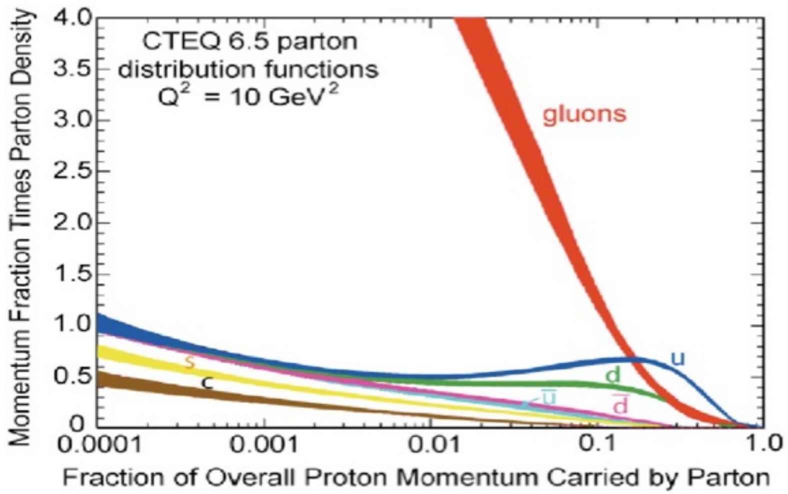
$$Q_s^2 \propto xg(x, Q_s)/R^2$$

Expectation: $x=10^{-5}$ in a proton at $Q^2=5$ GeV^2

If every term is equally important, counting single parton is meaningless – new state of saturated gluons: $\sigma^{\rm DIS}$ **stops to grow as** $x \rightarrow 0$

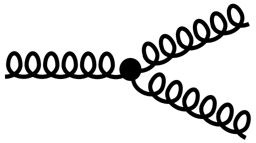
Gluon Saturation – Color Glass Condensate

❑ Run away gluon density at small-x?



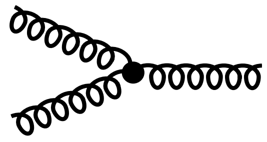
What causes the low-x rise?

gluon radiation
– non-linear gluon interaction



What could tame the low-x rise?

gluon recombination
– non-linear gluon interaction



❑ QCD vs. QED:

QCD – gluon in a proton:

$$Q^2 \frac{d}{dQ^2} xG(x,Q^2) \approx \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dx'}{x'} x' G(x',Q^2)$$

QED – photon in a positronium:

$$Q^2 \frac{d}{dQ^2} x\phi_\gamma(x,Q^2) \approx \frac{\alpha_{em}}{\pi} \left[-\frac{2}{3} x\phi_\gamma(x,Q^2) + \int_x^1 \frac{dx'}{x'} x' [\phi_{e^+}(x',Q^2) + \phi_{e^-}(x',Q^2)] \right]$$

✧ **At very small-x, proton is “black”, positronium is still transparent!**

✧ **Recombination of large numbers of glue could lead to saturation phenomena**

In the dipole model:

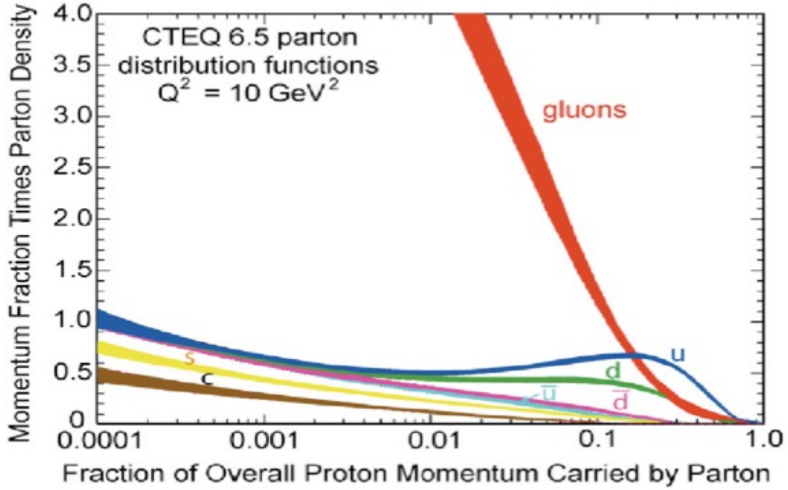
$$\frac{\partial N(x,k^2)}{\partial \ln(1/x)} = \alpha_s \mathcal{K}_{\mathrm{BFKL}} \otimes N(x,k^2) - \alpha_s \left[N(x,k^2) \right]^2$$

BK Equation

Jefferson Lab

Gluon Saturation – Color Glass Condensate

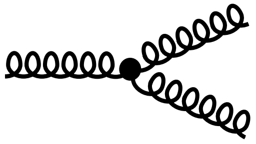
❑ Run away gluon density at small-x?



What causes the low-x rise?

gluon radiation

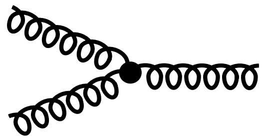
– non-linear gluon interaction



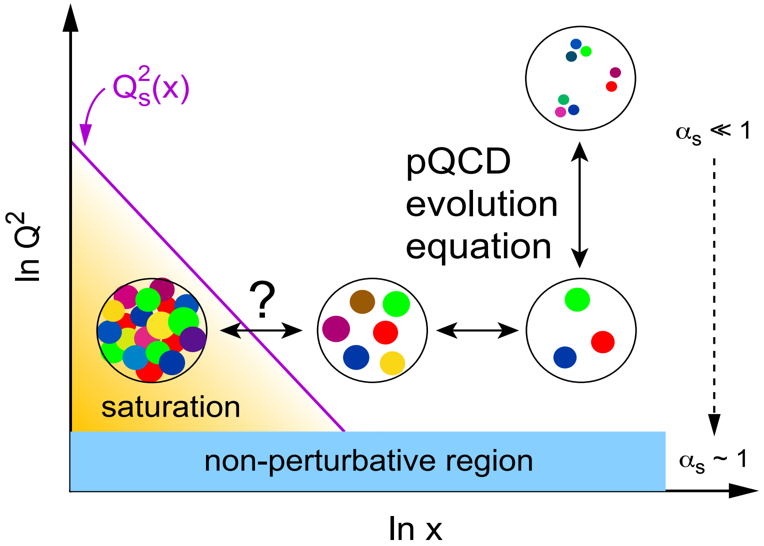
What could tame the low-x rise?

gluon recombination

– non-linear gluon interaction

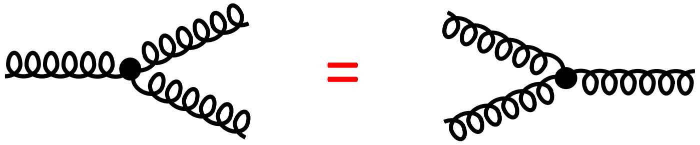


❑ Particle vs. wave feature:



Gluon saturation – Color Glass Condensate

Radiation = Recombination



at Q_s

Leading to a collective gluonic system?

with a universal property of QCD?

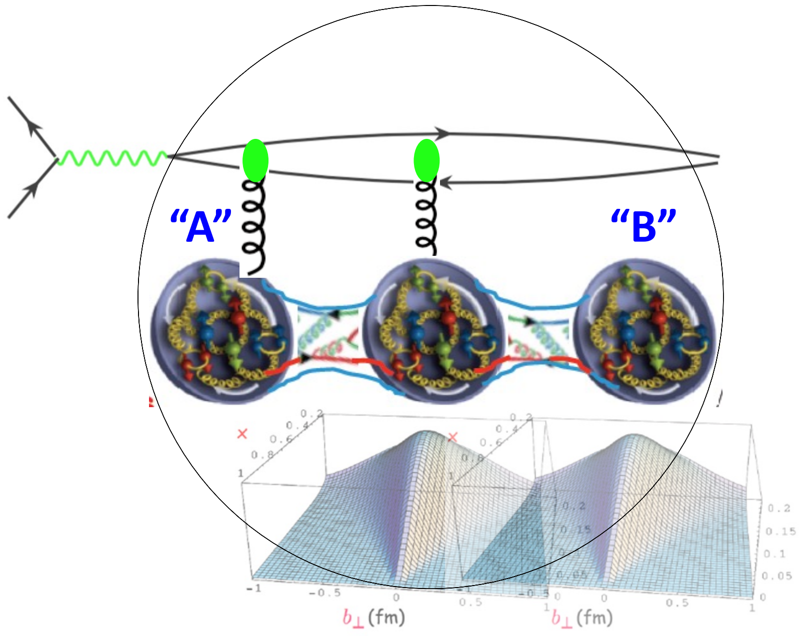
new effective theory QCD – CGC?

Small-x Physics in a large Nucleus

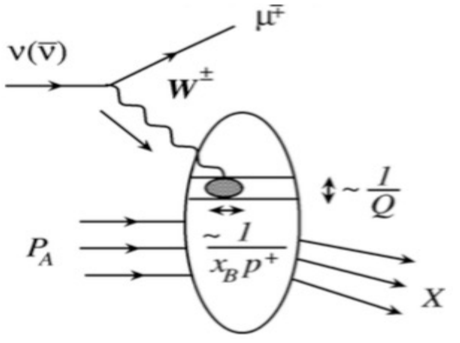
- A simple, but fundamental, question:

  What does a nucleus look like *if we only see quarks and gluons ?*

  Need localized hard probes – "see" more particle nature of the "glue"



- But, a hard probe at small-x is NOT necessarily localized:



In c.m. frame

Longitudinal probing size

> Lorentz contracted nucleon

if $$\frac{1}{xp} > 2R\frac{m}{p} \quad \text{or} \quad x < 0.1$$

A hard probe at small-x can interact with multiple nucleons (partons from multiple nucleons) at the same impact parameter coherently

- Another simple, and fundamental, question:

  Does the color of a parton in nucleon "A" know the color of another parton in nucleon "B"?

  IF YES, Nucleus could act like a bigger proton at small-x (long range of color correlation), and could reaching the saturation much sooner!

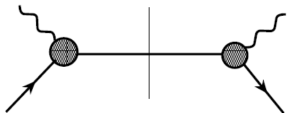
  IF NOT, only short-range color correlation, and observed nuclear effect in cross-section at small-x is dominated by coherent collision effect

Saturation of gluons is a part of QCD, where to find it? *EIC can tell !*

Jefferson Lab

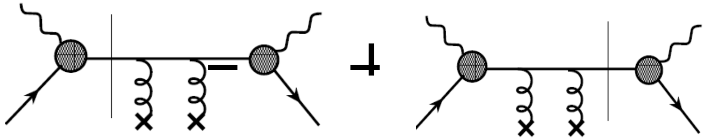
Multiple scattering in a large nucleus in DIS:

- **LO contribution to DIS cross section:**



$$\Rightarrow \delta(x - x_B)$$

- **NLO contribution:**

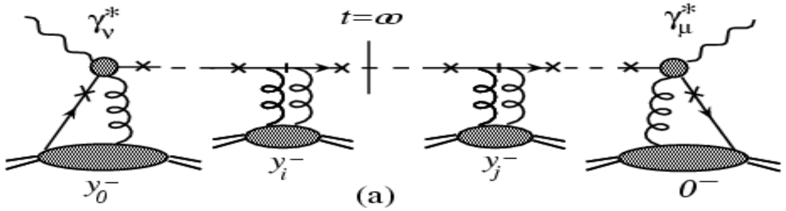


$$\Rightarrow \frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right] x_B \lim_{x_1\to x}\left[\frac{1}{x-x_1}\delta(x_1 - x_B) + \frac{1}{x_1 - x}\delta(x - x_B)\right]$$

$$2\pi^2\tilde{F}^2(0) \leftarrow \int \frac{dy_2^- dy_1^-}{(2\pi)^2}\left[F^{+\alpha}(y_2^-)F_\alpha{}^+(y_1^-)\right]\theta(y_2^-)$$

$$\lim_{x_1\to x}\left[\ldots\right] \to x_B\left[-\frac{d}{dx}\delta(x - x_B)\right]$$

- **Nth order contribution:**



$$\Rightarrow \left[\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]\right]^N x_B^N \lim_{x_i\to x}\sum_{m=0}^{N}\delta(x_m - x_B)\left[\prod_{i=1}^{m}\left(\frac{1}{x_{i-1} - x_m}\right)\right]\left[\prod_{j=1}^{N-m}\left(\frac{1}{x_{m+j} - x_m}\right)\right]$$

$$\to x_B^N\left[(-1)^N\frac{1}{N!}\frac{d^N}{dx^N}\delta(x - x_B)\right]$$

Infrared safe!

Multiple scattering in a large nucleus in DIS:

❑ Nuclear structure function:

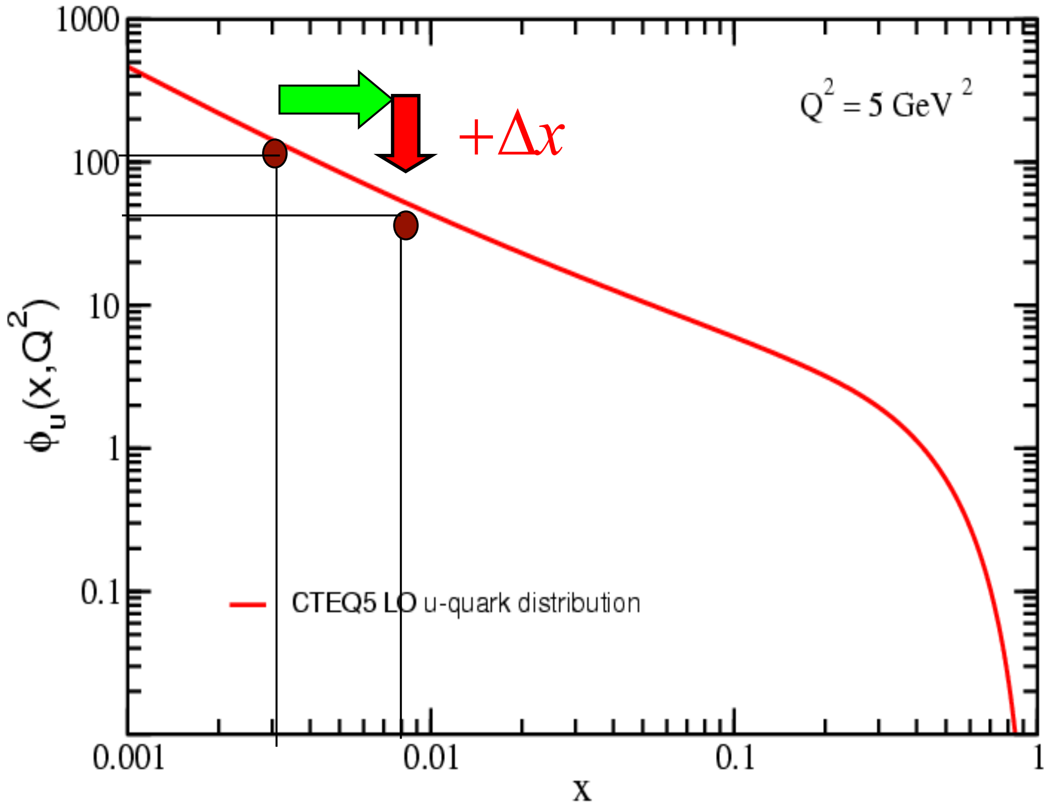
$$F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[\frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)$$

$$\approx F_T^{(0)}(x_B(1+\Delta), Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right)$$

$$\xi^2 = \frac{3\pi\alpha_s}{8R^2} \langle F^{+\alpha} F_\alpha{}^{+} \rangle$$

Single parameter for the power correction, and is proportional to the same characteristic scale



➡ Naturally lead to suppression of "cross section" if small-x coherent interaction if relevant!

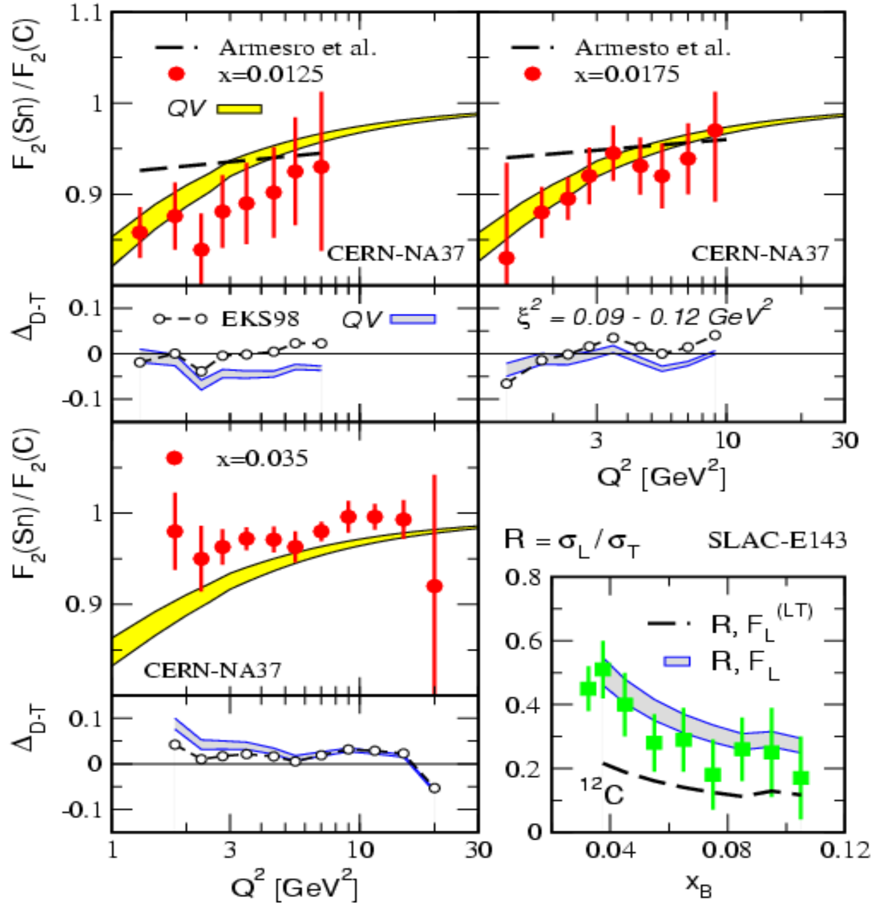
Similar result for longitudinal structure function

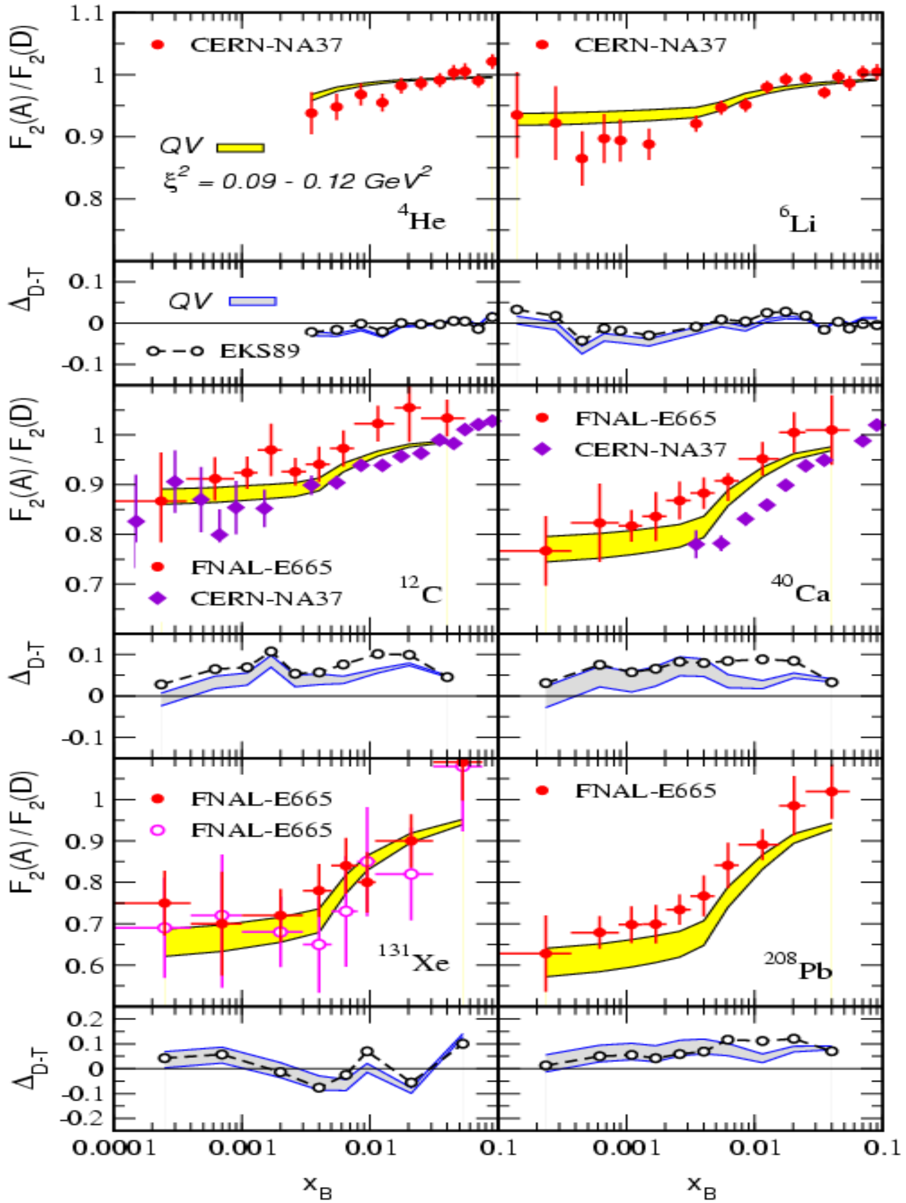
Multiple scattering in a large nucleus in DIS:

- **Broadening parameter:**

$$\xi^2 \sim 0.09 - 0.12 \text{ GeV}^2$$

From A-dependence of Jet broadening exp't





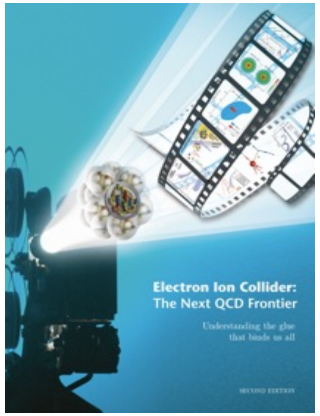
One number for all x_B, Q, and A dependence !

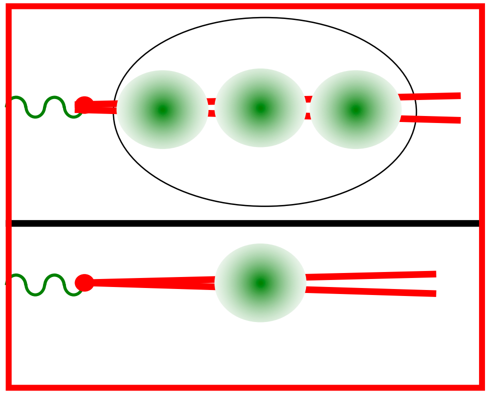
Coherent Length of the Color

- A simple experiment to address a "simple" question:

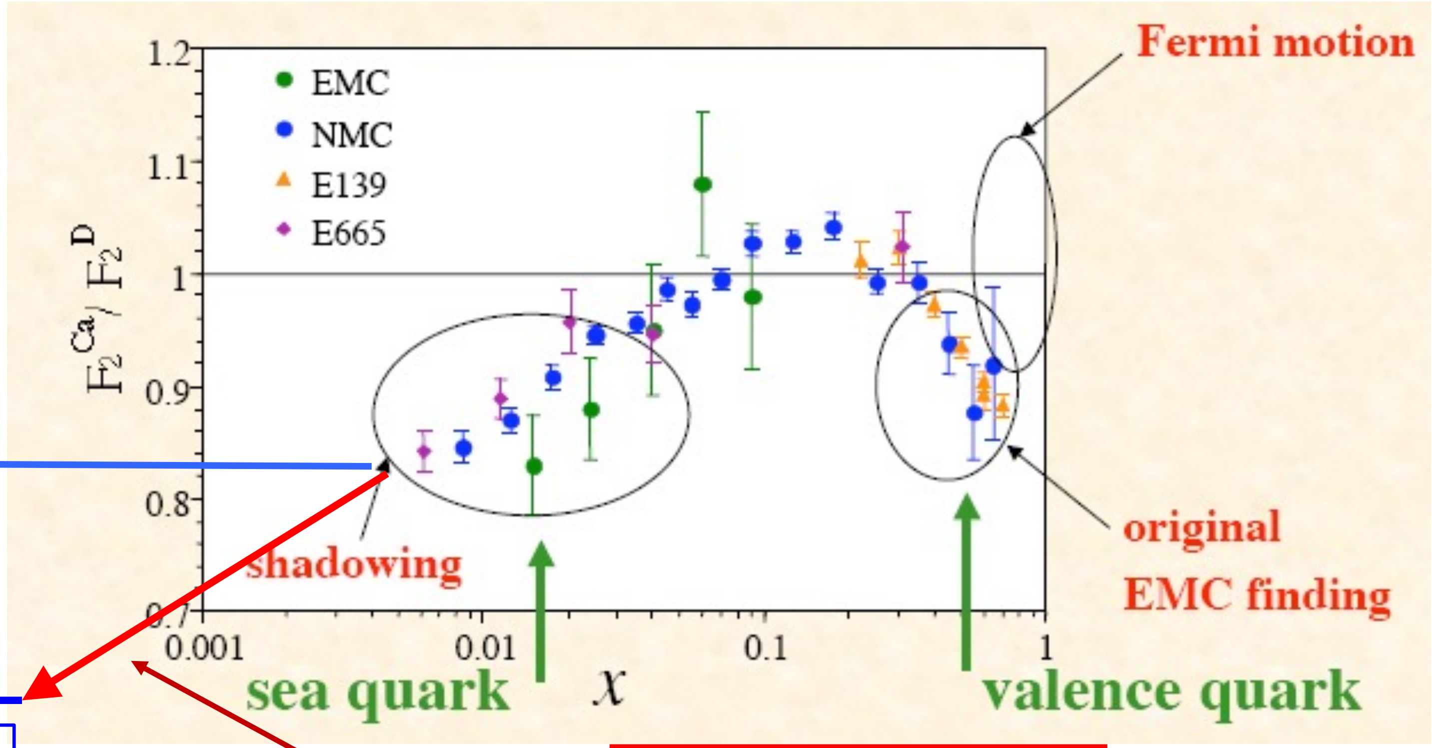
Will the nuclear shadowing continue to fall as x decreases?

EIC White Paper



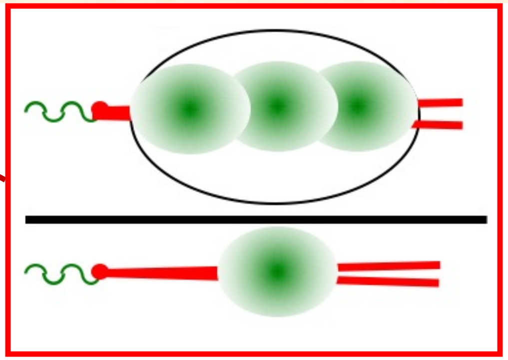


Color localized
Inside nucleons



Nucleus as a
bigger proton

Color leaks outside nucleons
Proton radius of soft gluon is larger !



EIC can tell !

Summary and Outlook

- **We have the right Theory – QCD, but, unprecedented challenges**
  - QCD has been very successful in describing the short-distance dynamics
  - Trying to understand the emergent phenomena of QCD:
    - Hadron properties, such as the mass, spin, …, in the most fundamental way
    - Internal structure and landscape of hadrons, such as confined motion, spatial tomography of nuclei, …
    - Emergence of hadrons from quarks and gluons, neutralization of the color, femto-meter sized detectors, …
    - Particle and wave nature of quarks and gluons, …

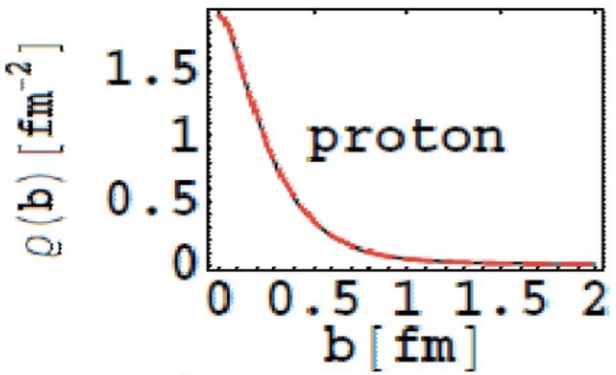
- **EIC is an ultimate QCD machine and a facility, capable of discovering and exploring the emergent phenomena of QCD, and the role of color and glue, …, and the science of Nuclear Femtography**

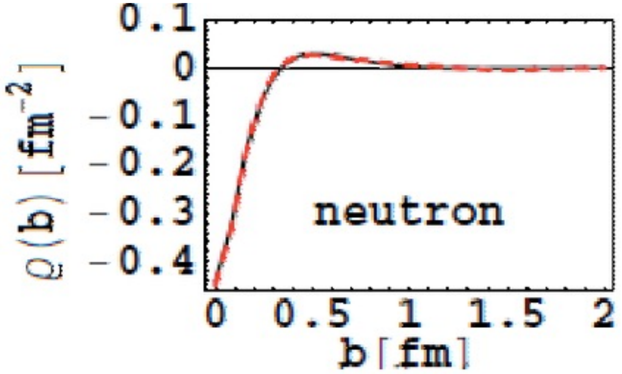
- **US-EIC is sitting at a sweet spot for rich QCD dynamics, capable of taking us to the next frontier of QCD and the Standard Model!**

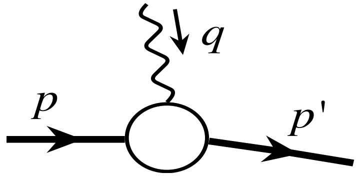
Thanks!

Explore Internal Structure of Hadron without Breaking it

- **Form factors:** Elastic electric form factor → Charge distributions

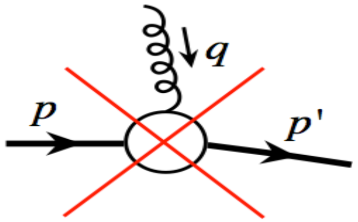






Proton "Radius"
in EM charge distribution

- **But, there is NO elastic "color" form factor!**



No Proton "Radius"
in color charge distribution!

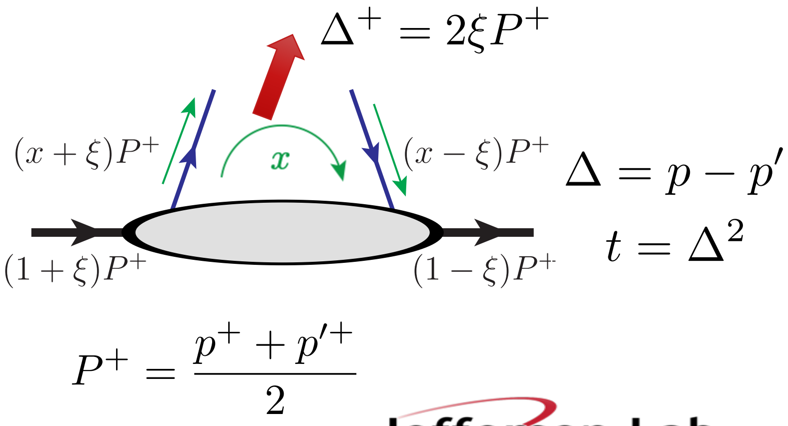
- **Combine PDF and Form Factor – GPDs:**

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$



Similar definition for gluon GPDs