The Electron-Ion Collider (EIC)

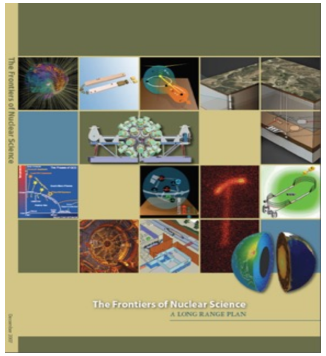
- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Emergent Properties and Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons without breaking them?
  – *Spin as a tool to select*
- Lec. 4: Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

Jianwei Qiu
Theory Center, Jefferson Lab

U.S. - based Electron-Ion Collider (EIC)

- **A long journey, a joint effort of the full community:**

…




The EIC Science case: a report on the joint BNL/INT/JLab program

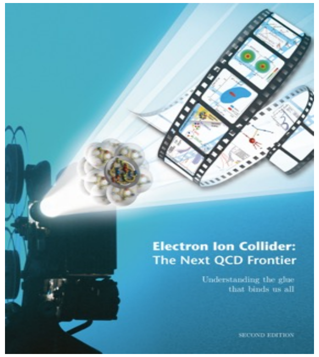
Gluons and the quark sea at high energies: distributions, polarization, tomography

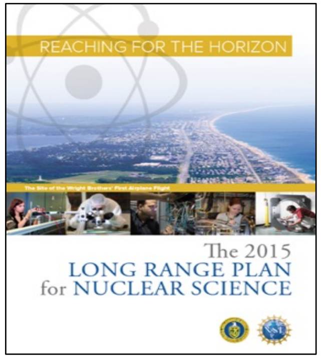
Institute for Nuclear Theory, University of Washington, USA
September 13 to November 19, 2010

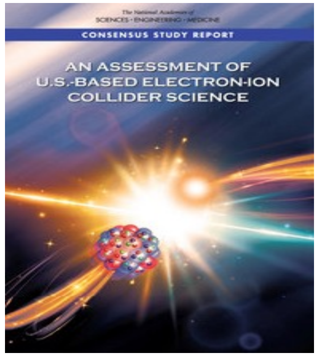
Editors:
D. Boer, Universiteit Groningen, The Netherlands
M. Diehl, Deutsches Elektronen-Synchroton DESY, Germany
R. Milner, Massachusetts Institute of Technology, USA
R. Venugopalan, Brookhaven National Laboratory, USA
W. Vogelsang, Universität Tübingen, Germany

Brookhaven National Laboratory, Upton, NY
Institute for Nuclear Theory, Seattle, WA
Thomas Jefferson National Accelerator Facility, Newport News, VA

arXiv:1108.1713v2 [nucl-th] 28 Nov 2011







…

2018

"… answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics."

… three profound questions:

How does the mass of the nucleon arise?
How does the spin of the nucleon arise?
What are the emergent properties of dense systems of gluons?

- **January 9, 2020:** ***The U.S. DOE announced the selection of BNL as the site for the Electron-Ion Collider***
  July 6, 2021: **Achieved Critical Decision 1 (CD1) approval,**
  Hope to have CD2 in 2025, & an operational machine in 2035 (President's budget)

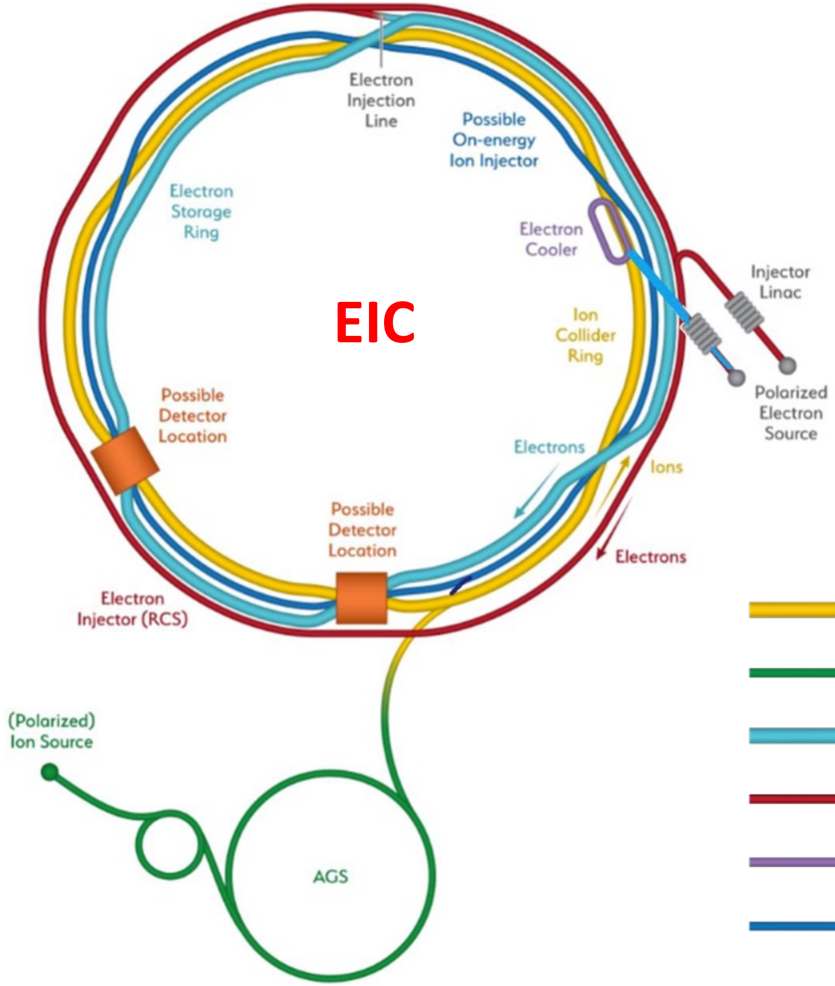
→ **A new era to explore the emergent phenomena of QCD!**

U.S. - based Electron-Ion Collider (EIC)

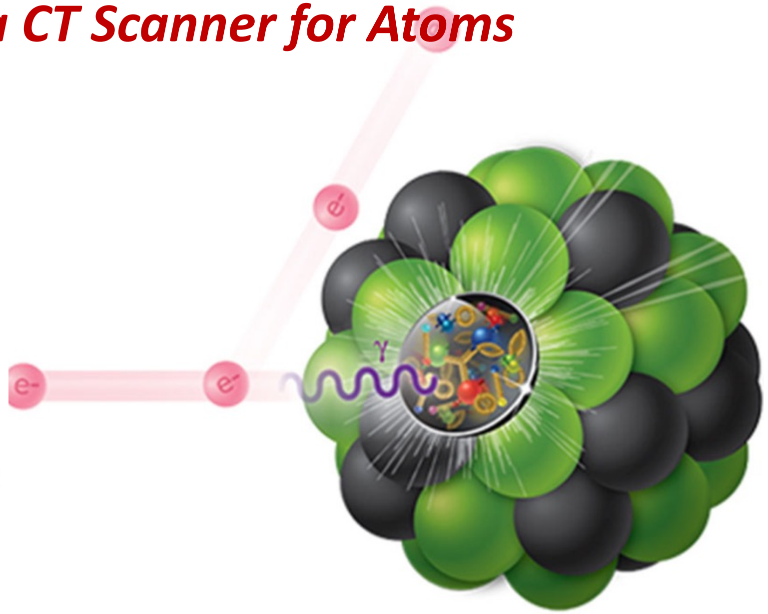
https://www.bnl.gov/eic/

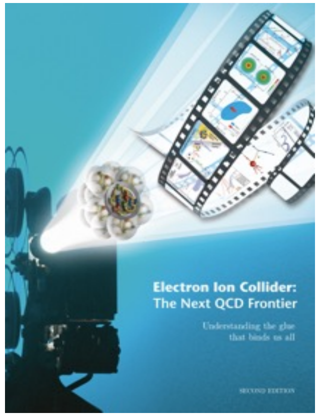
A machine that will unlock the secrets of the strongest force in Nature

Like a CT Scanner for Atoms



- Hadron Storage Ring
- Hadron Injector Complex
- Electron Storage Ring
- Electron Injector Synchrotron
- Electron Cooler
- Possible On-energy Hadron Injector Ring





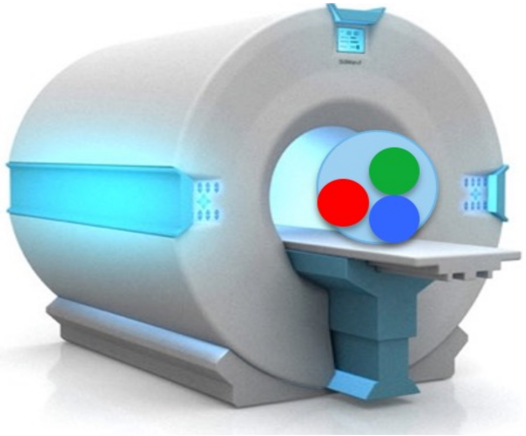
Basic Tech Requirements

- **Center of Mass Energies:**
  20 GeV – 141 GeV
- **Required Luminosity:**
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- **Hadron Beam Polarization:**
  80%
- **Electron Beam Polarization:**
  80%
- **Ion Species Range:**
  p to Uranium
- **Number of interaction regions:**
  up to two

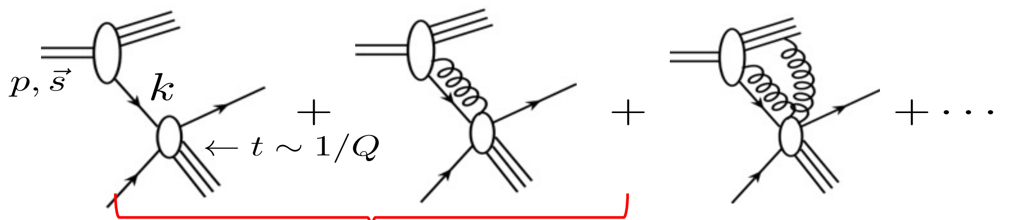
US-EIC – can do what HERA could not do

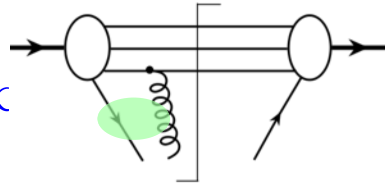
- **Quantum imaging:**
  - ✧ HERA discovered: 10-15% of e-p events is diffractive – Proton not broken!
  - ✧ US-EIC: 100-1000 times luminosity – *Critical for 3D tomography!*

Large momentum transfer without breaking the proton
Luminosity!



- **Quantum interference & entanglement:**
  - ✧ US-EIC: Highly polarized beams – *Origin of hadron property: Spin, …*
    Direct access to chromo-quantum interference!

$\sigma(Q, \vec{s}) \propto \Big|$  $\Big|^2$

$\sigma(s) - \sigma(-s)$ ➡ **Quantum interference** ➡ $T^{(3)}(x, x) \propto$ 

No probability interpretation!

- **Nonlinear quantum dynamics:**
  - ✧ US-EIC: Light-to-heavy nuclear beams – *Origin of nuclear force, …*
    Catch the transition from chromo-quantum fluctuation to chromo-condensate of gluons, …
    Emergence of hadrons (nuclei as femtometer size detectors!),
    – "a new controllable knob" – Atomic weight of nuclei

Wave nature of quark/gluon field

Frontiers of QCD and Strong Interaction

- Understanding where did we come from?

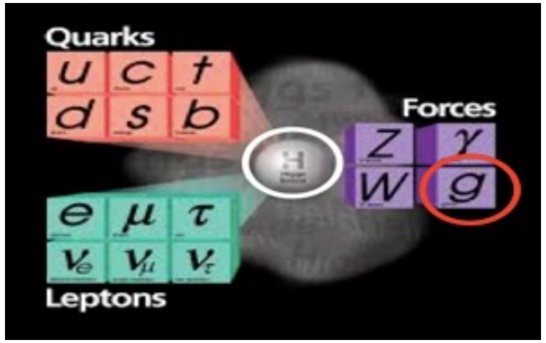
Global Time: →

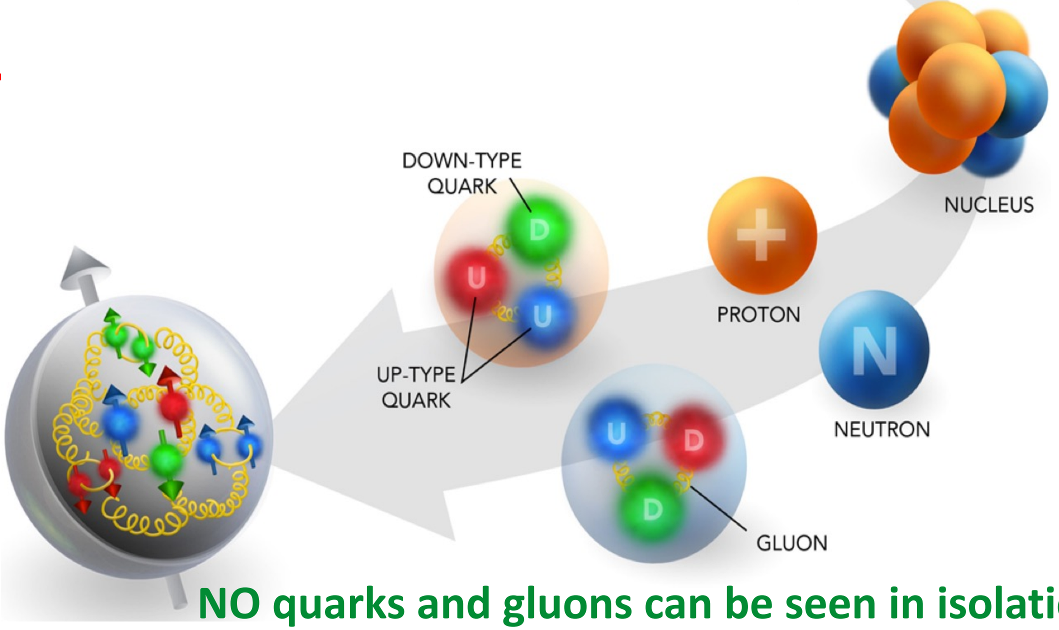
See Helen's lectures

QCD at high temperature, high densities, phase transition, …

Facilities – Relativistic heavy ion collisions: SPS, RHIC, the LHC, …

- Understanding what are we made of?





NO quarks and gluons can be seen in isolation!

- Try to understand the emergent properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?
- Understanding QCD fully is still beyond the best mind that we have!

Nuclear Femtography:

Search for answers to these questions at a Fermi scale!

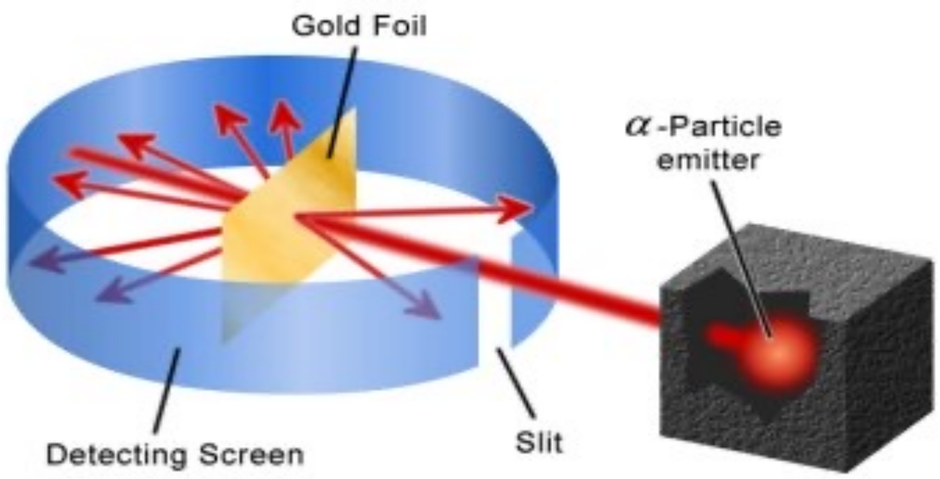
Facilities – CEBAF, EIC, EICC, LHeC, …

Jefferson Lab

How to See Internal Structure of a Hadron – Breaking it?

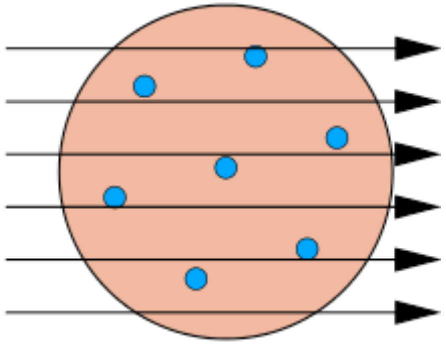
❑ Atomic structure: dating back to Rutherford experiment (over 100 years ago):

Experiment setup: $\alpha + Au \rightarrow \alpha + X$



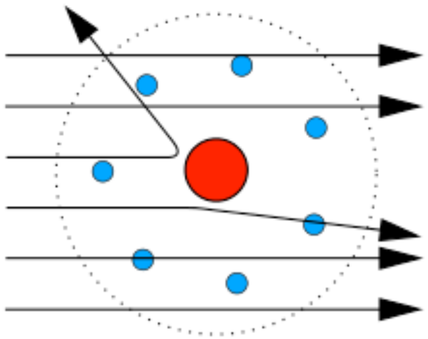
Only measure the scattered α–particle
Energy and scattering angle

Expectation



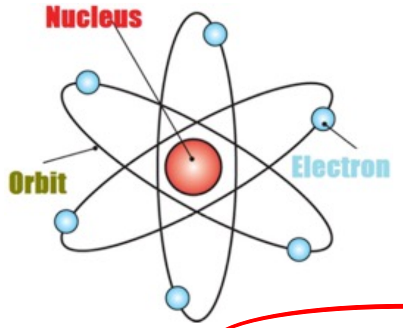
J.J. Thomson's plum-pudding model

Discovery



Rutherford's Experiment - Data

Theory

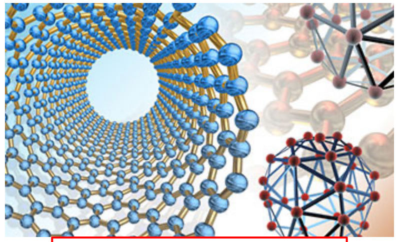


Quantum orbitals

Discovery:

- **Tiny nucleus** – *less than 1 trillionth in volume of an atom*
- **Quantum probability** – *the new Quantum World!*

➡ ***Infinite opportunities to create & improve !*** ➡



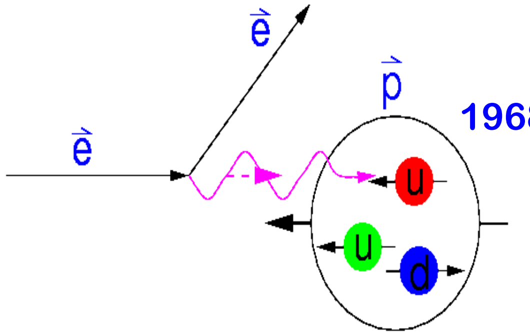
Nano-science (1-100 nm)

How to See Internal Structure of a Hadron – Breaking it?

❑ A modern "Rutherford" experiment (over 50 years ago):

SLAC Deep Inelastic Scattering (DIS)

$$e(l) + h(p) \to e'(l') + X$$



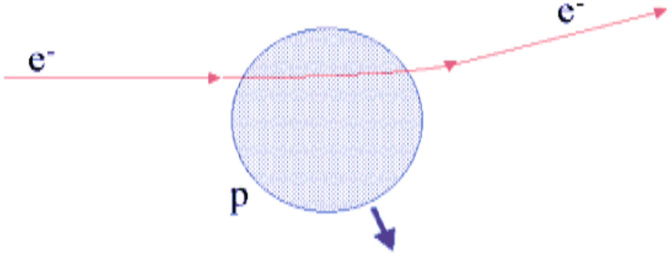
with a localized probe:

$$Q^2 = -(l - l')^2 \gg 1\ \text{fm}^{-2}$$

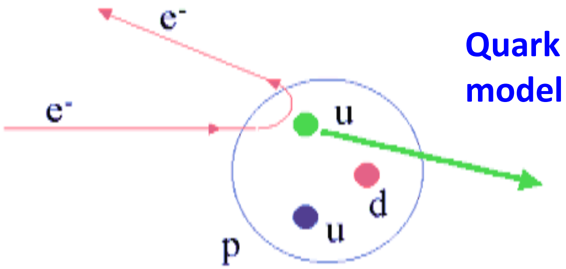
$$\frac{1}{Q} \ll 1\ \text{fm}$$

Prediction

◆ If proton "charge cloud":

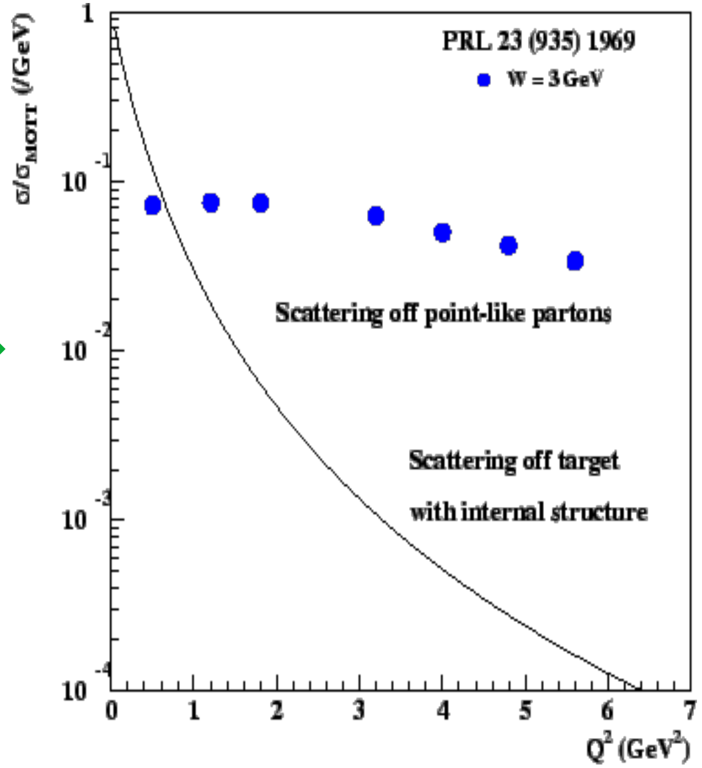


◆ If proton contains point charges, some of time see:



Quark model

Discovery



Only measure the scattered lepton Energy and scattering angle

Discovery:

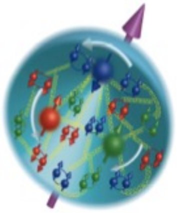
- **Partons/Quarks – Electric charged, spin-1/2 particles**
- **Led to the birth of Quantum Chromodynamics (QCD) – gluons & color force!**

Jefferson Lab

From SLAC experiment to the Parton Model

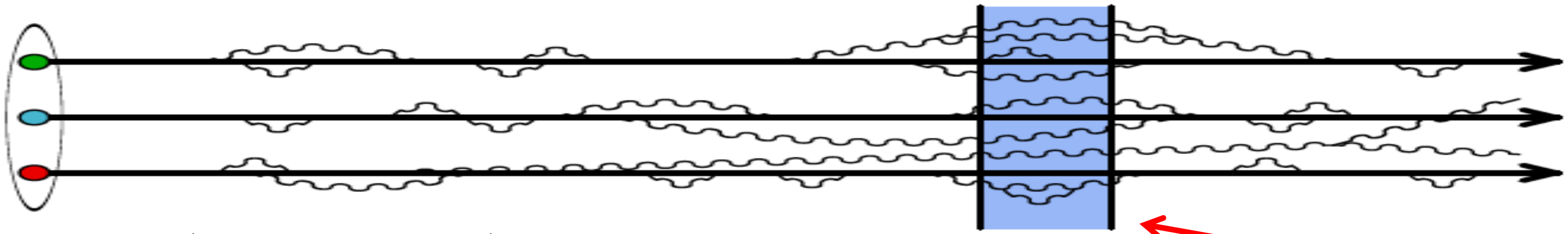
❑ Feynman's parton picture :

High energy scattering with a large momentum transfer: $Q \gg 1/R \sim 1/\text{fm} \sim 200\ \text{MeV}$



p

Boost



Time-dilation

$k^\mu = (k^+, k^-, k_T) \approx (k^+, 0^-, 0_T)$

$k^+ = xp^+ \sim Q \gg \langle k^- \rangle,\ \langle k_T \rangle$ ➡ $k^\mu \approx xp^\mu$

Hard probe (t ~ 1/Q << fm)

Feynman:

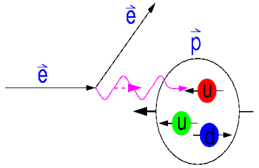
At t ~ 1/Q << fm, the hard probe is only sensitive to the momentum fraction of the probed "parton" (quark or gluon) xP ~ Q >> k_T, and the probability f(x) to find this "parton" (quark or gluon)

➡ ***f(x) = Probability to "catch" the quantum fluctuation!***

Momentum fraction: $x = k^+/p^+$

❑ Feynman's parton model for DIS:

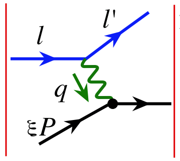
$$\sigma_{\text{DIS}}(x, Q^2) = \left| \quad \right|^2$$



➡
$$d\hat{\sigma}_{ei\to e'X} = \frac{1}{2\hat{s}} \left|\mathcal{M}_{ei\to e'X}\right|^2 \times (2\pi)^4 \delta(l + xp - l' - k) \times \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3l'}{(2\pi)^3 2E'}$$

➡
$$E' \frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'} \propto \delta(\xi - x)$$

$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} = \sum_i \int d\xi\, f_{i/h}(\xi)\, E' \frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'}$$



➡
$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} \propto \sum_i e_i^2 f_{i/h}(x)$$

Quantum Chromodynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:**

$\psi_i^f(x)$ Quark fields: spin-½ Dirac fermion (like electron)

Color triplet: $i = 1, 2, 3 = N_c$

Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$ Gluon fields: spin-1 vector field (like photon)

Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c} \right]^2 + \text{gauge fixing} + \text{ghost terms}$$

- **QED – force to hold atoms together:**

$$\mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}^f \left[(i\partial_\mu - eA_\mu)\gamma^\mu - m_f \right] \psi^f - \frac{1}{4}\left[\partial_\mu A_\nu - \partial_\nu A_\mu \right]^2$$

QCD is much richer in dynamics than QED

Gluons are dark, but, interact with themselves, NO free quarks and gluons

Gauge Properties of QCD:

Gauge Invariance:

$$\psi_i(x) \to \psi'_j(x) = U(x)_{ji}\,\psi_i(x)$$
$$A_\mu(x) \to A'_\mu(x) = U(x)\,A_\mu(x)\,U^{-1}(x) + \frac{i}{g}\,[\partial_\mu U(x)]\,U^{-1}(x)$$

where $A_\mu(x)_{ij} \equiv A_{\mu,a}(x)(t_a)_{ij}$

$$U(x)_{ij} = \left[e^{i\,\alpha_a(x)\,t_a}\right]_{ij} \qquad \text{Unitary} \quad [\text{det=1, SU(3)}]$$

Color matrices:

$$[\,t_a, t_b\,] = i\,C_{abc}\,t_c$$

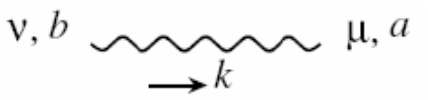
Generators for the fundamental representation of SU3 color

Gauge Fixing:

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}(\partial_\mu A_a^\mu)(\partial_\nu A_a^\nu)$$

Allow us to define the gauge field propagator:

$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$



with $\lambda = 1$ **the Feynman gauge**

Ghost in QCD

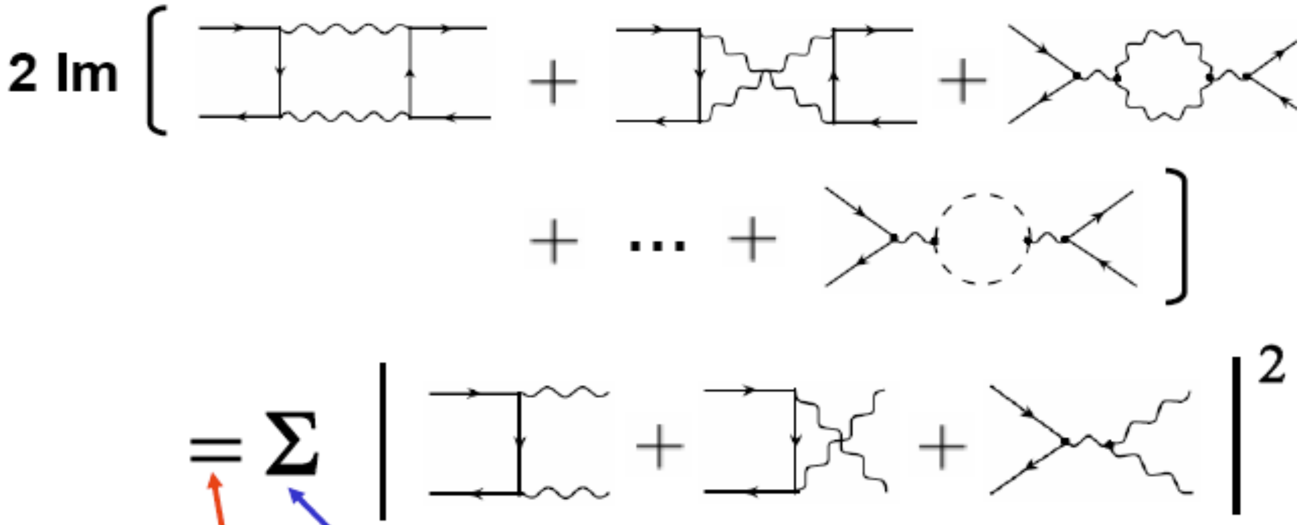
- **Ghost:**

Only needed when we are working in a covariant gauge

Ghost

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a(x))(\partial^\mu \eta_a(x) - g\, C_{abc}\, A_b^\mu(x)\, \eta_c(x))$$

so that the optical theorem (hence the unitarity) can be respected,



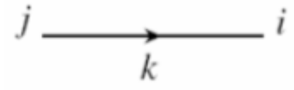
Sum over all physical polarizations

Fail without the ghost loop

Feynman Rules in QCD

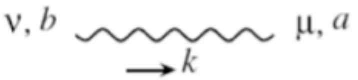
Propagators:

Quark:



$$\frac{i}{\gamma \cdot k - m}\,\delta_{ij}$$

Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

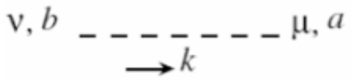
for a covariant gauge

$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n}\right]$$

for a light-cone gauge

$$n \cdot A(x) = 0 \quad \text{with} \quad n^2 = 0$$

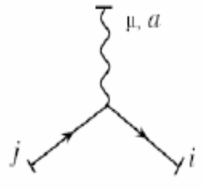
Ghost:



$$\frac{i\delta_{ab}}{k^2}$$

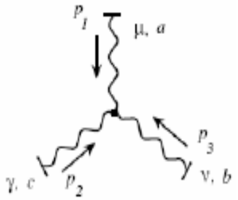
Interactions:

$$-g\bar{\psi}\gamma^\mu A_{\mu,a} t_a \psi$$



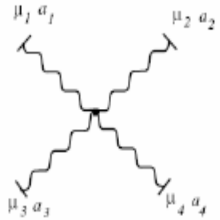
$$-ig(t_a)_{ij}\gamma_\mu$$

$$\frac{1}{2} g C_{abc}(\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a}) A_b^\mu A_c^\nu$$



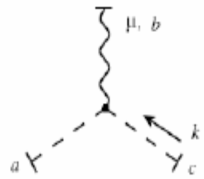
$$-gC_{abc}\left[g_{\mu\nu}(p_1 - p_2)_\gamma + g_{\nu\gamma}(p_2 - p_3)_\mu + g_{\gamma\mu}(p_3 - p_1)_\nu\right]$$

$$-\frac{g^2}{4} C_{abc} C_{ab'c'} * A_b^\mu A_c^\nu A_{\mu,b'} A_{\nu,c'}$$



$$-ig^2\left[C_{ea_1a_2} C_{ea_3a_4} * (g_{\mu_1\mu_3} g_{\mu_2\mu_4} - g_{\mu_1\mu_4} g_{\mu_2\mu_3}) + \ldots\right]$$

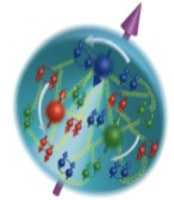
$$\partial_\mu \bar{\eta}_a \left(g C_{abc} A_b^\mu\right) \eta_c$$



$$g C_{abc} k_\mu$$

Only needed when we are working in a covariant gauge

QCD Color is Fully Entangled



❑ QCD color confinement:

- *Do not see any quarks and gluons in isolation*
- *The structure of nucleons and nuclei – emergent properties of QCD*

Color Confinement — Asymptotic freedom — Q (GeV)

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale

QCD at the Fermi Scale: Femto-science (0.1-10 fm)

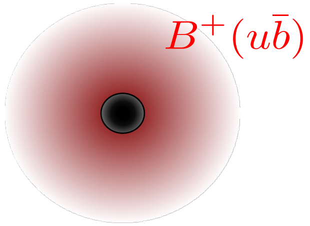
- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

Asymptotic regime PQCD works beautifully!

❑ QCD is non-perturbative:

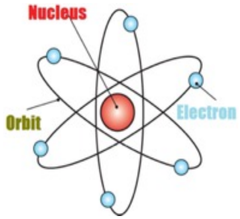
- *Any cross section/observable with identified hadron is not perturbatively calculable!*
- *Color is fully entangled!*

B-meson



Brown-Muck

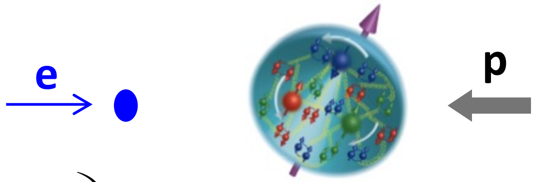
Atomic structure

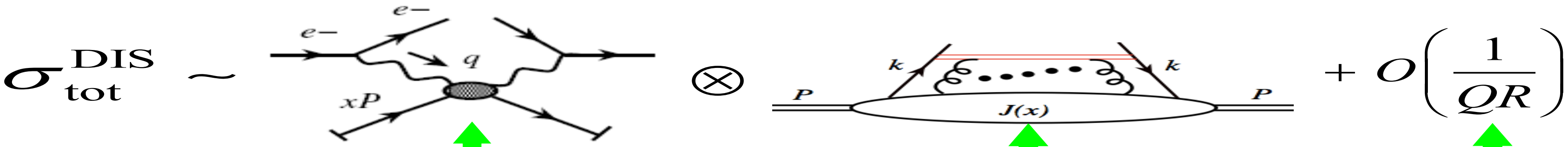


Quantum orbits

Theoretical Approaches – Approximations:

- **Perturbative QCD Factorization:**
  - *– Approximation at Feynman diagram level*





Probe Hard-part	Structure Parton-distribution	Approximation Power corrections

- **Effective field theory (EFT):**
  - *– Approximation at the Lagrangian level*

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD:**
  - *– Approximation for finite lattice spacing, finite box, lightest quark masses, … with Euclidean time formulation (removable with increased computational cost)*

  Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

- **Other approaches:**

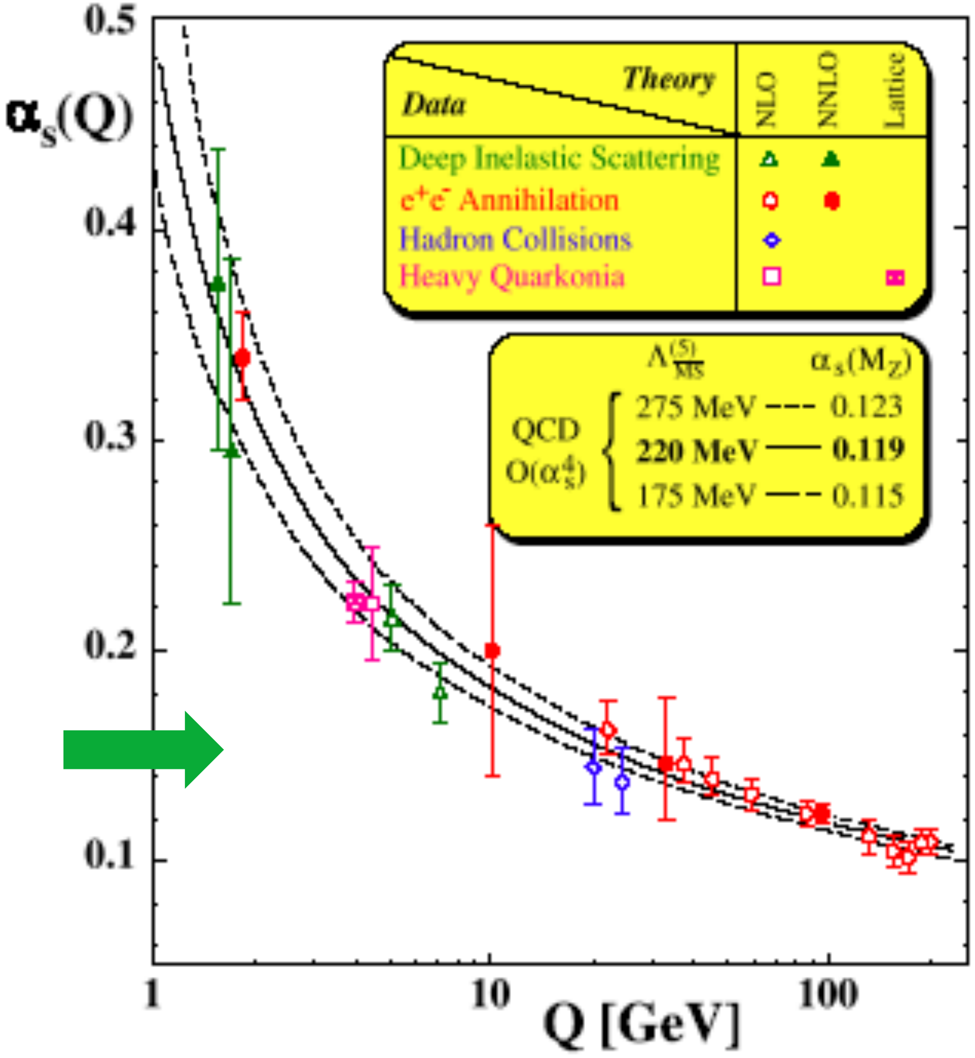
  Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

QCD Asymptotic Freedom

- **Interaction strength:**

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{QCD}^2}\right)}$$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

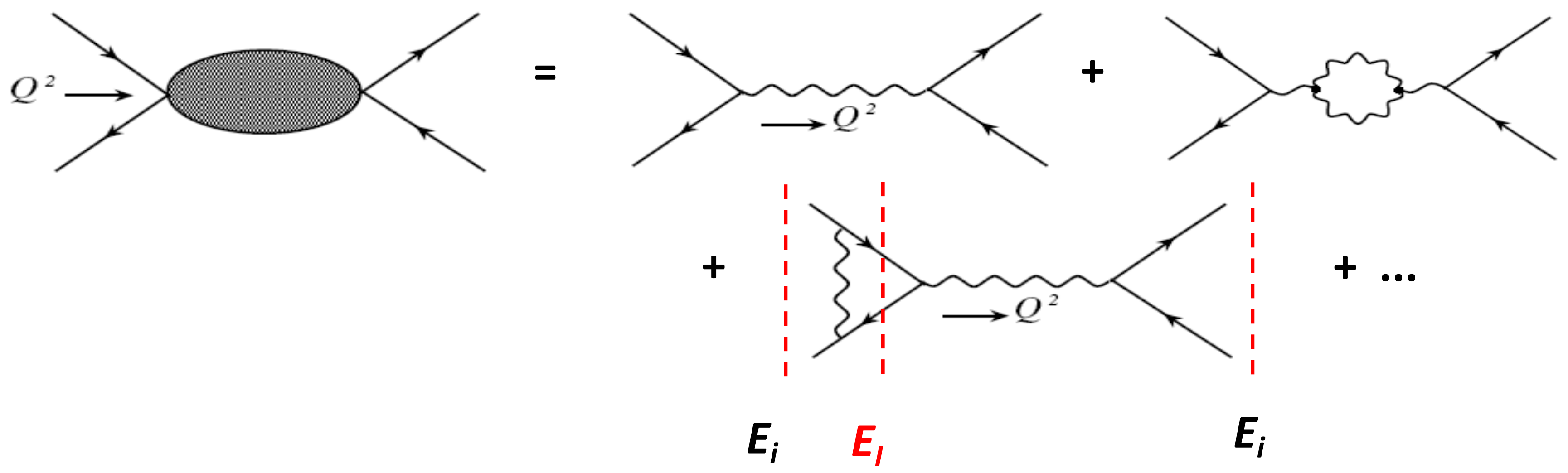
2004 Nobel Prize in Physics

Discovery of QCD Asymptotic Freedom

Controllable perturbative QCD calculations at HIGH ENERGY!

Renormalization, Why need?

❑ Scattering amplitude:



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \ldots \right) + \ldots \Rightarrow \infty$$

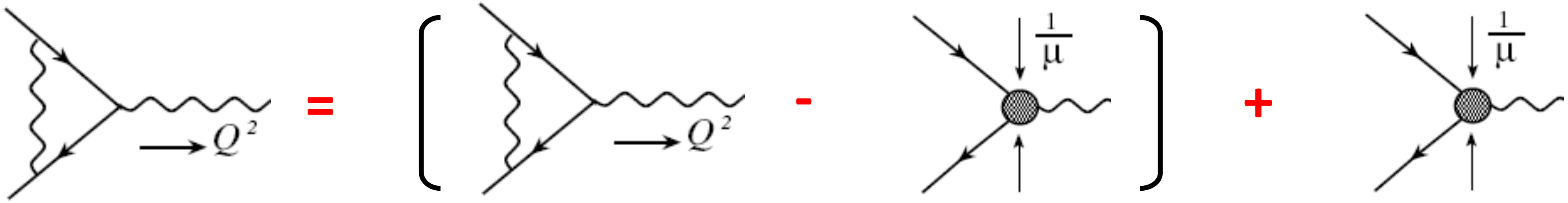
UV divergence: **result of a "sum" over states of high masses**

Uncertainty principle: **High mass states = "Local" interactions**

No experiment has an infinite resolution!

Physics of Renormalization

- UV divergence due to “high mass” states, not observed

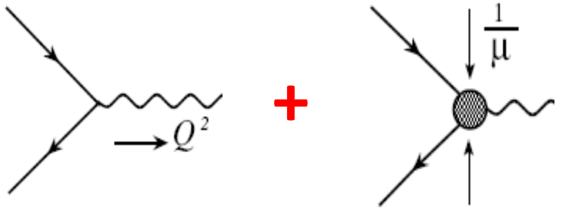


“Low mass” state

“High mass” states

- Combine the “high mass” states with LO

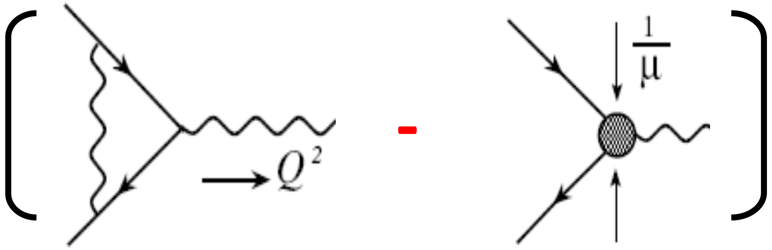
LO:



$= g(\mu)$

Renormalized coupling

NLO:



+ …

No UV divergence!

- Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantity should not depend on the choice of renormalization scale μ
  ➡ renormalization group equation:

$$\mu^2 \frac{d}{d\mu^2}\, \sigma_{\text{Phy}}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0 \quad \Longrightarrow \quad \sigma_{\text{Phy}}(Q^2) = \sum_n \hat{\sigma}^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- Running coupling constant:

$$\mu \frac{\partial g(\mu)}{\partial \mu} = \beta(g) \qquad \alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- QCD β function:

$$\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu} = +g^3 \frac{\beta_1}{16\pi^2} + \mathcal{O}(g^5) \qquad \beta_1 = -\frac{11}{3} N_c + \frac{4}{3}\frac{n_f}{2} < 0 \quad \text{for } n_f \leq 6$$

- QCD running coupling constant:

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1) \ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \quad \text{as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Effective Quark Mass

- **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since } \gamma_m(g(\lambda)) > 0$$

- **Choice of renormalization scale:**

$\mu \sim Q$ **for small logarithms in the perturbative coefficients**

- **Light quark mass:**

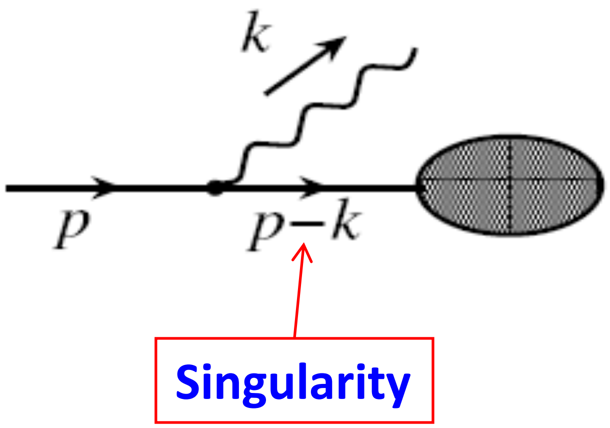
$$m_f(\mu) \ll \Lambda_{\rm QCD} \quad \text{for } f = u, d, \text{ even } s$$

QCD perturbation theory ($Q \gg \Lambda_{QCD}$)
is effectively a massless theory

Infrared and Collinear Divergences

❑ **Consider a general diagram:**

$$p^2 = 0, \quad k^2 = 0$$ **for a massless theory**



✧ $$k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$$

⇨ **Infrared (IR) divergence**

✧ $$k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1$$

$$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2\, p^2 = 0$$

⇨ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

Infrared Safety (IRS)

❑ Infrared safety:

$$\sigma_{\text{Phy}}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right) + \mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = $\kappa > 0$

Purely perturbative calculations alone (exploiting asymptotic freedom) are only useful for quantities that are infrared safe (IRS)!

❑ Cross section with identified hadron(s):

- ***Can not be calculated perturbatively!***
- ***Solution – QCD factorization:***
  - ***to isolated what can be calculated perturbatively,***
  - ***to represent the leading non-perturbative information by universal functions***
  - ***to justify the approximation to neglect other nonperturbative information,*** *such as power corrections, …*

Foundation of QCD Perturbation Theory

- **Renormalization**
  - QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- **Asymptotic freedom**
  - weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Welczek

- **Infrared safety and factorization**
  - calculable short distance dynamics
  - pQCD factorization – connect the partons to physical cross sections

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe and factorizable observables!

Physical Observables

Cross sections with identified hadron(s)
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is NOT
a perturbative scale

Look for two-types physical observables:

- ❑ **Purely infrared safe quantities**
- ❑ **Observables with identified hadron(s), but, factorizable in QCD**

Fully Infrared Safe Observables – I

Fully inclusive, without any identified hadron!

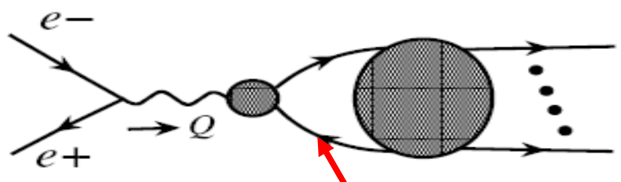
$$\sigma^{\text{total}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{total}}_{e^+e^-\to\text{partons}}$$

The simplest observable in QCD

e^+e^- ➔ Hadrons Inclusive Cross Sections

- ❑ e^+e^- ➔ hadron **total** cross section – not a specific hadron!

$$\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} \propto \left| \quad \right|^2$$



Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} \propto \sum_n {\rm P}_{e^+e^-\to n} = \sum_n \sum_m {\rm P}_{e^+e^-\to m} {\rm P}_{m\to n} = \sum_m {\rm P}_{e^+e^-\to m} \sum_n {\rm P}_{m\to n} \quad = 1$$

$$\sigma^{\rm tot}_{e^+e^-\to \rm partons} \propto \sum_m {\rm P}_{e^+e^-\to m} \quad \Rightarrow \quad \sigma^{\rm tot}_{e^+e^-\to \rm hadrons} = \sigma^{\rm tot}_{e^+e^-\to \rm partons}$$

- ❑ e^+e^- ➔ parton **total** cross section:

$$\sigma^{\rm tot}_{e^+e^-\to \rm partons}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

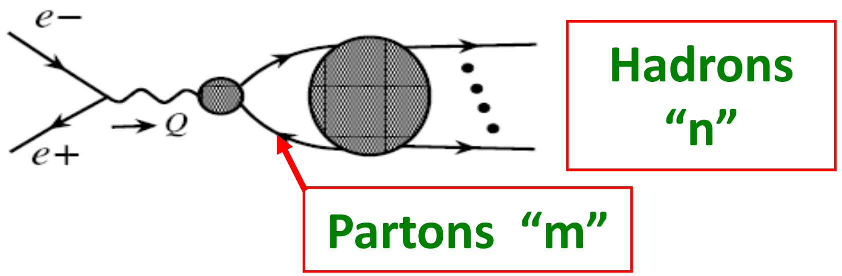
Finite in perturbation theory – KLN theorem

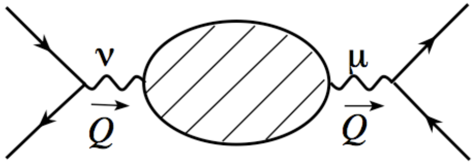
Calculable in pQCD

Infrared Safety of e^+e^- Total Cross Sections

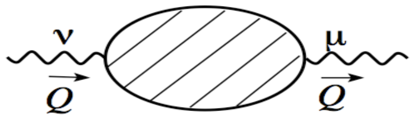
❑ Optical theorem:

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto \left| \quad \right|^2 \propto \text{Im}$$





❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right) \Pi(Q^2)$$

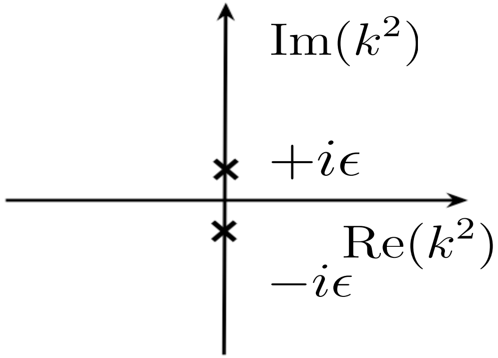
IR safety of $\sigma^{\text{tot}}_{e^+e^-\to\text{partons}}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

❑ IR safety of $\Pi(Q^2)$:

If there were pinched poles in $\Pi(Q^2)$,

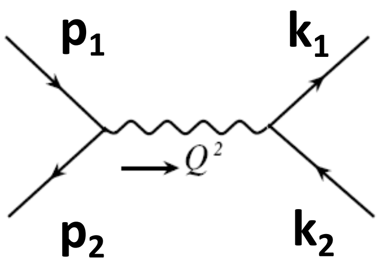
- ✧ **real partons moving away from each other**
- ✧ **cannot be back to form the virtual photon again!**

Rest frame of the virtual photon



Lowest Order (LO) Perturbative Calculation

- **Lowest order Feynman diagram:**



- **Invariant amplitude square:**

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]$$

$$s=(p_1+p_2)^2$$
$$t=(p_1-k_1)^2$$
$$u=(p_2-k_1)^2$$

- **Lowest order cross section:**

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt}=\frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \quad \text{where } s=Q^2$$

$$\sigma_2^{(0)}=\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}=\sum_Q e_Q^2 N_c\frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

Threshold constraint

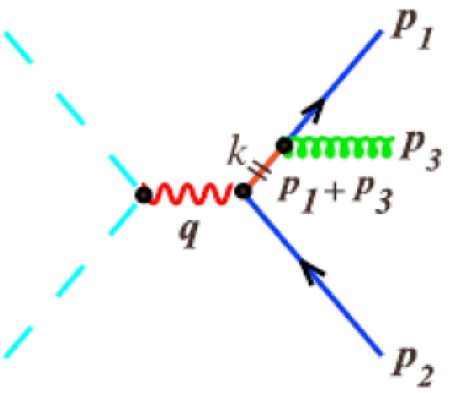
One of the best tests for the number of colors

Jefferson Lab

Next-to-Leading Order (NLO) Contribution

- **Real Feynman diagram:**

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2 p_i . q}{s} \qquad \text{with } i = 1, 2, 3$$



+ crossing

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2 \qquad 2(1 - x_1) = x_2 x_3 (1 - \cos\theta_{23}), \quad cycl.$$

- **Contribution to the cross section:**

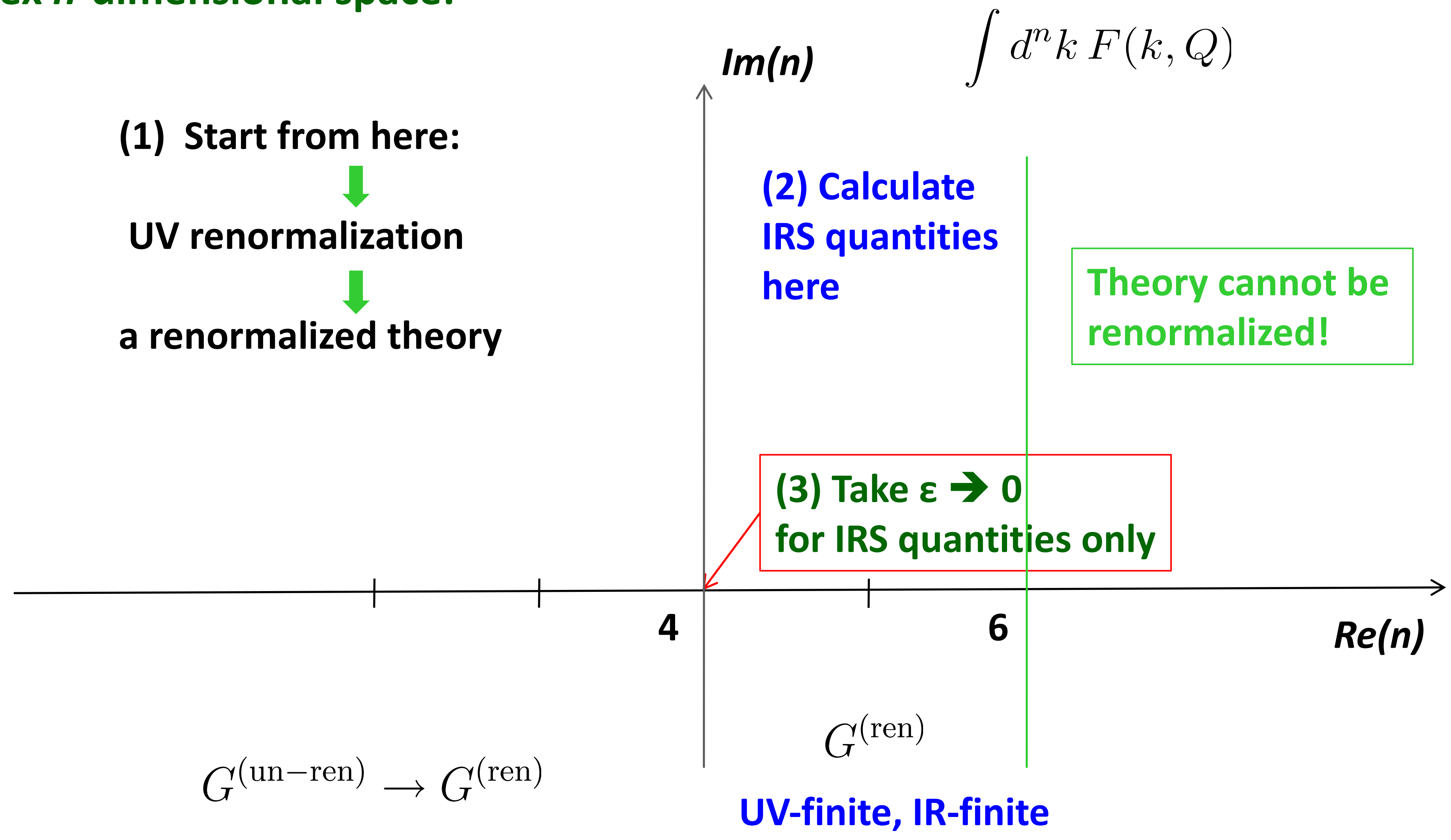
$$\frac{1}{\sigma_0} \frac{d\sigma_{e^+e^- \to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

IR as $x3 \to 0$
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

Divergent as $x_i \to 1$
Need the virtual contribution and a regulator!

How Does Dimensional Regularization Work?

- ❑ **Complex *n*-dimensional space:**



Dimensional Regularization for both IR and CO

❑ **NLO with a dimensional regulator:**

✧ **Real:**

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

✧ **Virtual:**

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

✧ **NLO:**

$$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi} + O(\varepsilon)\right]$$

No ε dependence!

✧ **Total:**

$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O(\alpha_s^2)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Highest order perturbative calculations

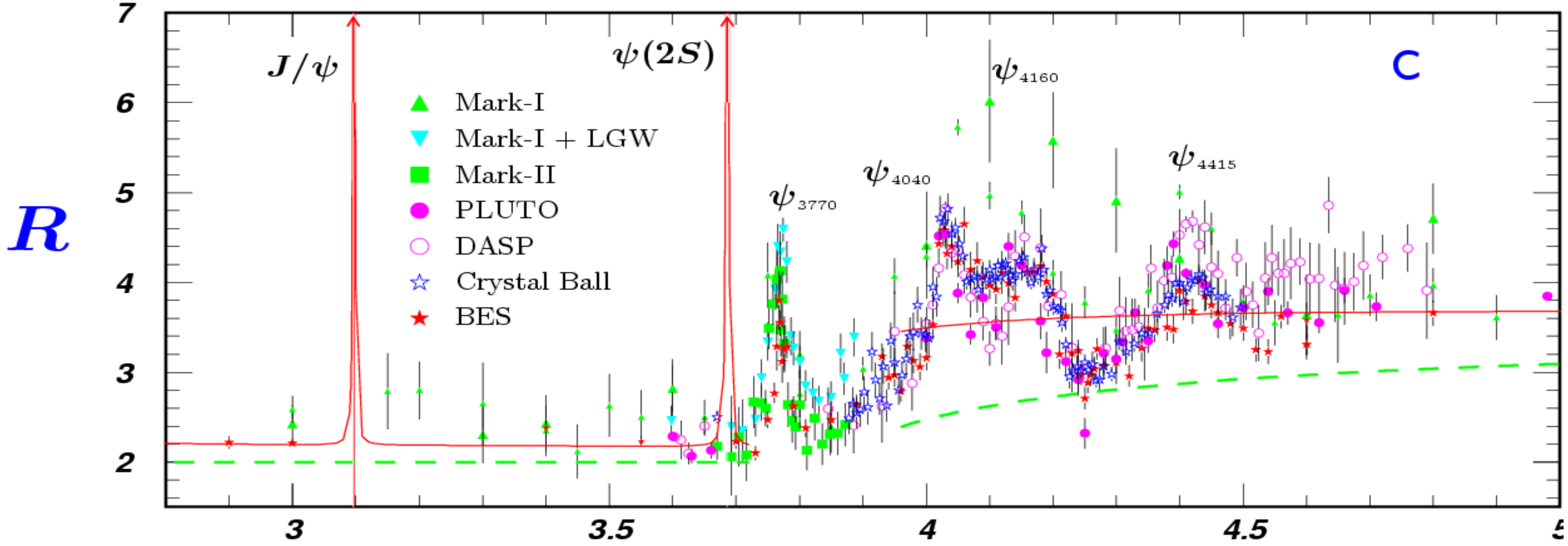
Hadronic Cross Section in e+e- Collisions

- Normalized hadronic cross section:

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$
$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1+\frac{\alpha_s(s)}{\pi}+\mathcal{O}(\alpha_s^2(s))\right]$$
$$+N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1+\frac{2m_q^2}{s}\right)\sqrt{1-\frac{4m_q^2}{s}}+\mathcal{O}(\alpha_s(s))\right]$$

$N_c = 3$

$$\rightarrow 2\left[1+\frac{\alpha_s(s)}{\pi}+\ldots\right]$$



Might still be the best observable for confirming N_c=3!

Go beyond the inclusive total cross section?

Fully Infrared Safe Observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\text{Jets}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{Jets}}_{e^+e^-\to\text{partons}}$$

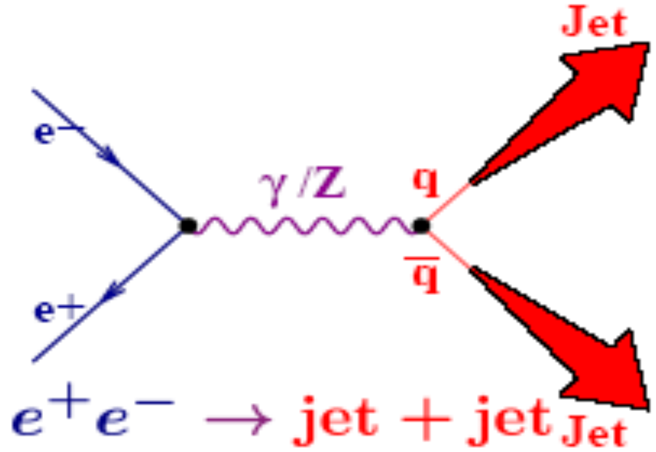
Jets – as the "trace" or "footprint" of partons

Thrust distribution in e^+e^- collisions

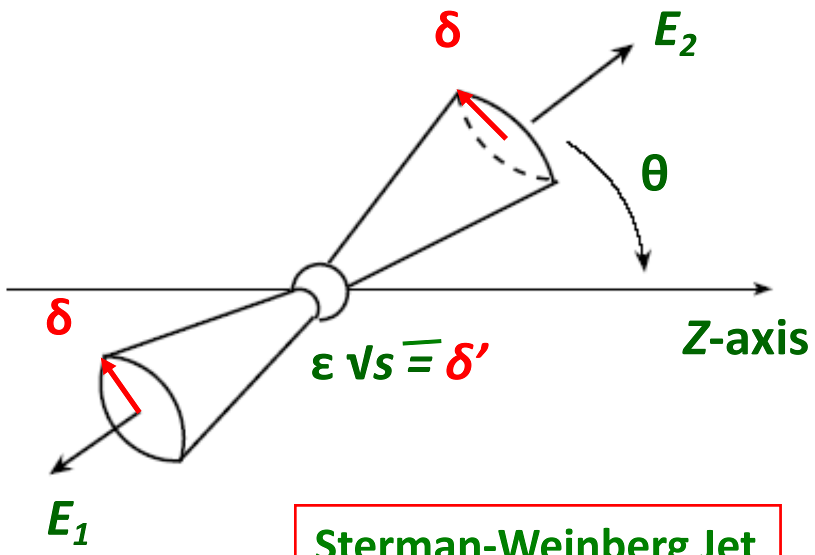
etc.

Jets – the Trace of Partons

- Jets – "total" cross-section
  with a limited phase-space
  Not any specific hadron!



- Q: will the IR cancellation be completed
  with the constraint on the phase space?
  - ✧ Leading partons are moving away from each other, carrying color!
  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a "jet"
    – "trace" of a parton



Sterman-Weinberg Jet

- Many Jet algorithms

Infrared Safety for Restricted Cross Sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under interchange of n-particles

- Conditions for IRS of dσ(Γ):

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with } 0 \le \lambda \le 1$$

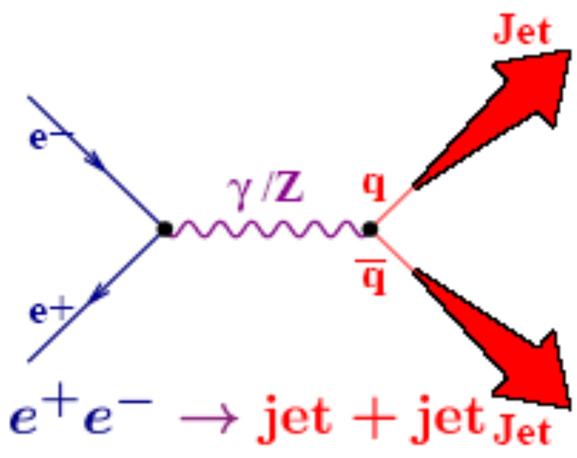
Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without this parton – inclusiveness!

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1 \text{ for all } n \quad \Rightarrow \quad \sigma^{(\text{tot})}$

An Early Clean Two-Jet Event

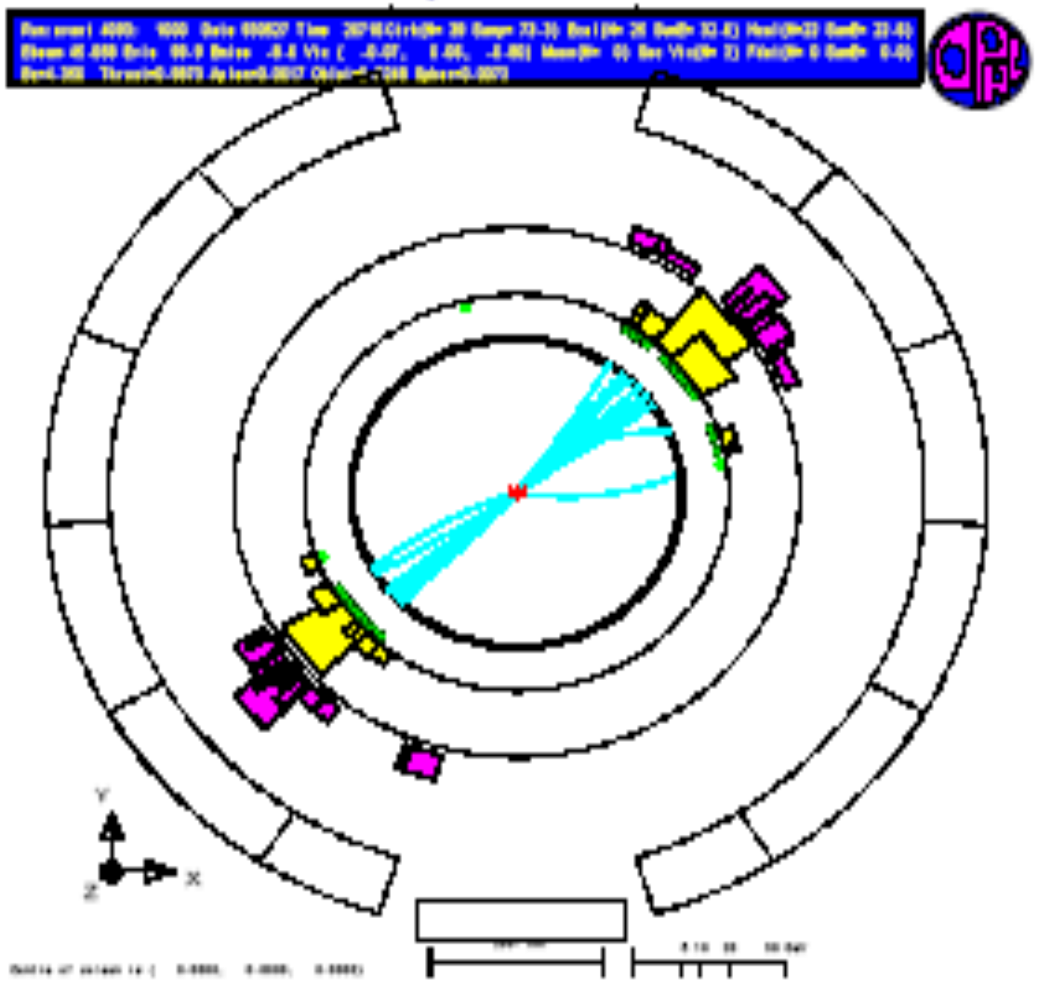
Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

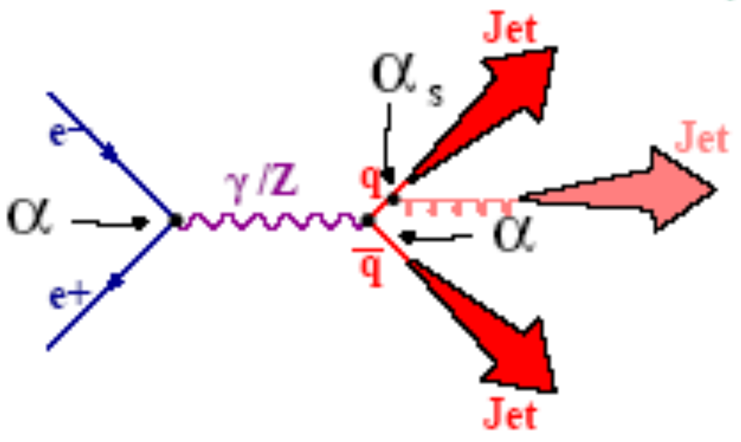
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



Early Three-Jet Event – Discovery of the Gluon or the Gluon Jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

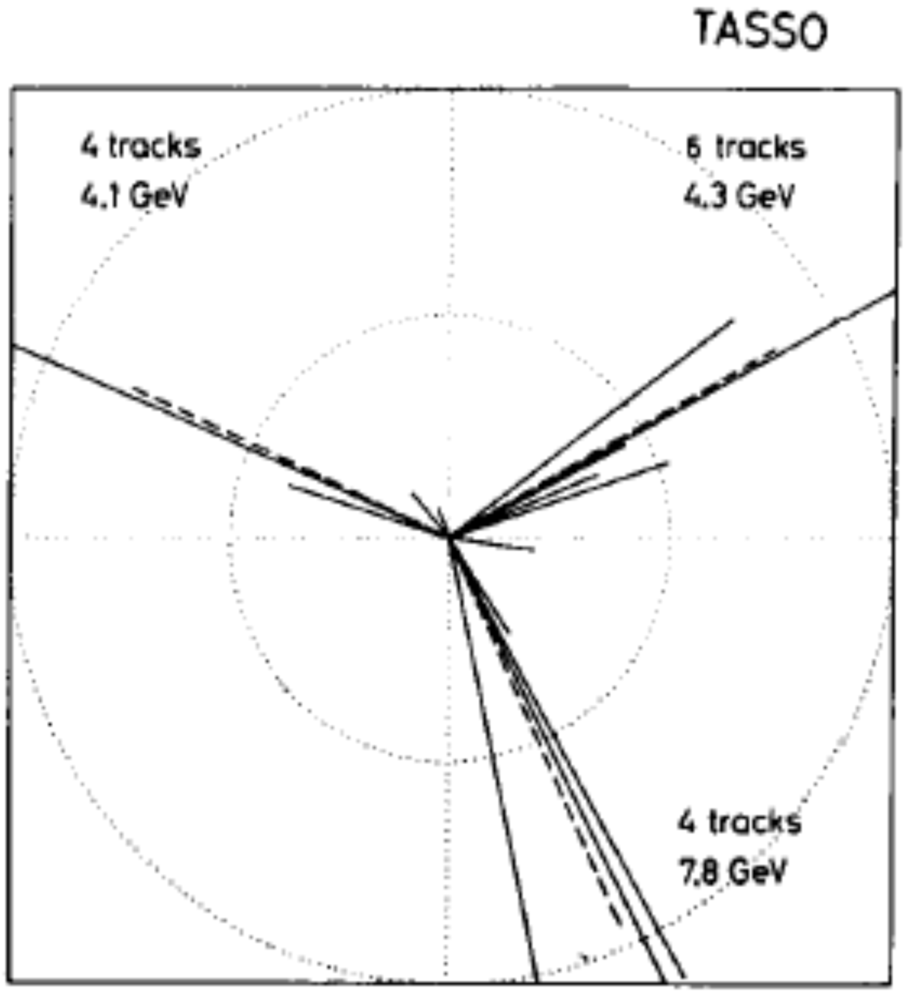
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

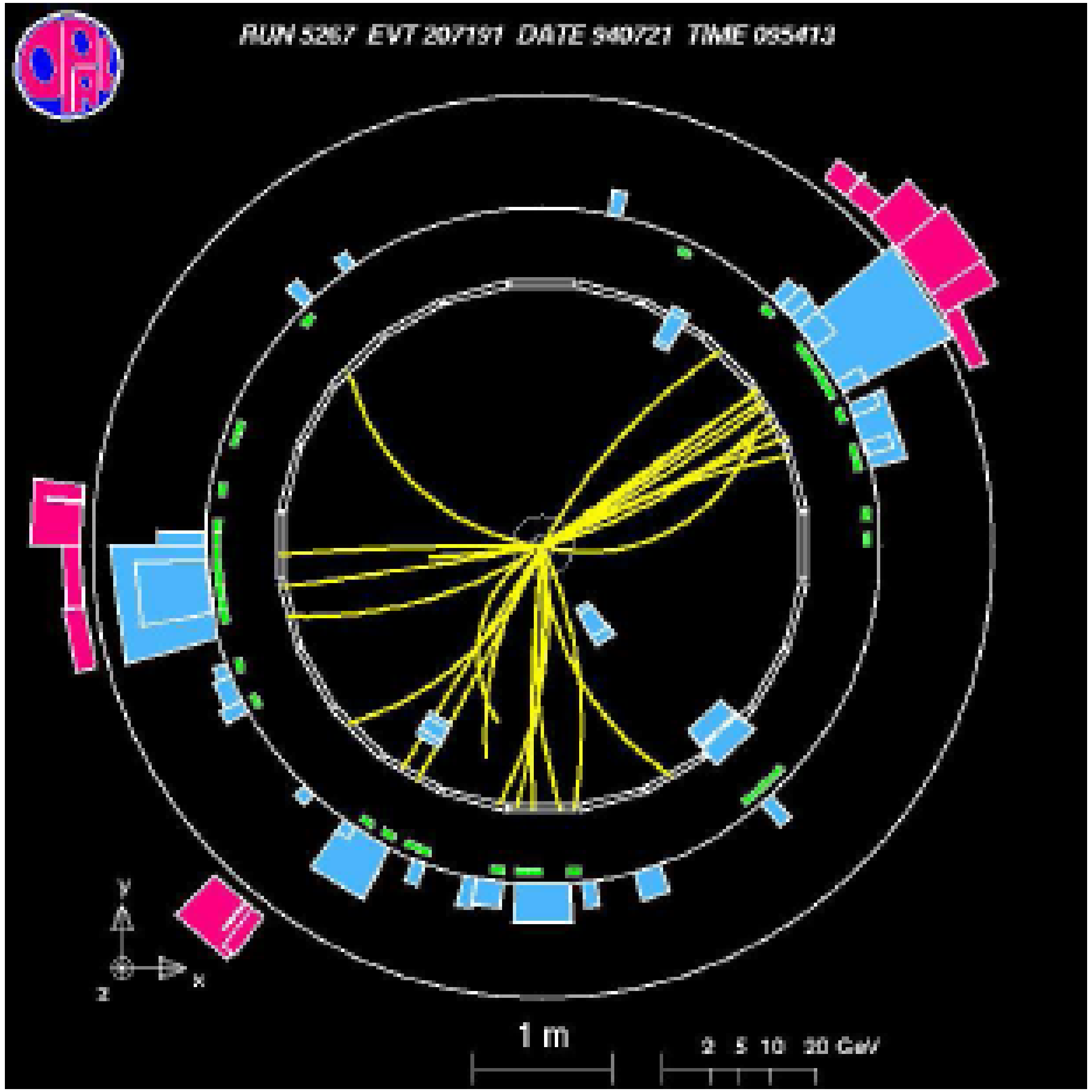
PETRA e^+e^- storage ring at DESY:

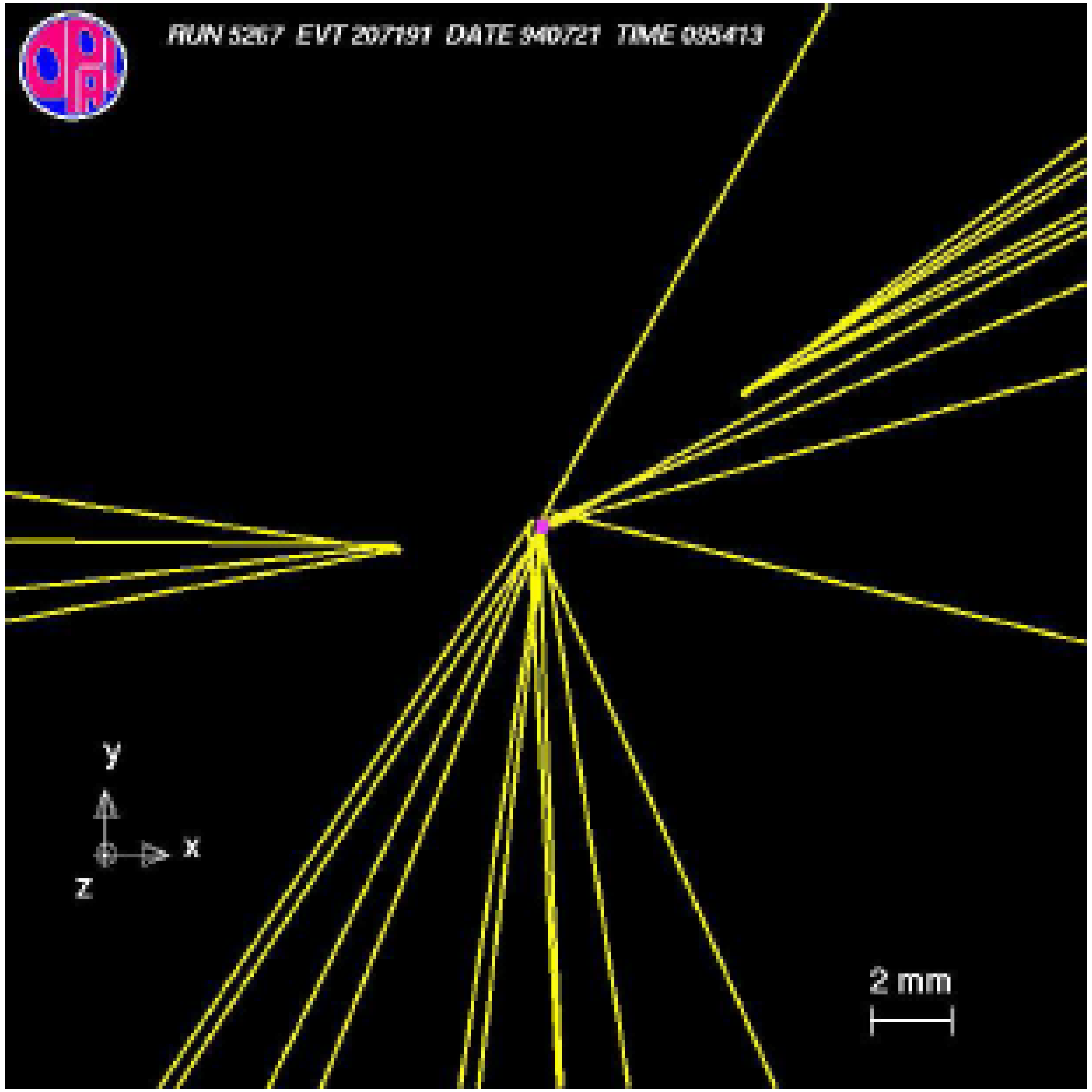
$E_{c.m.} \gtrsim 15$ GeV



Jefferson Lab

Tagged Three-Jet Event from LEP

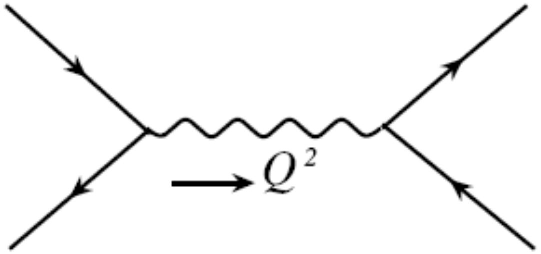




Two-Jet Cross Section in e+e- Collisions

- **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



- **Two-jet in pQCD:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

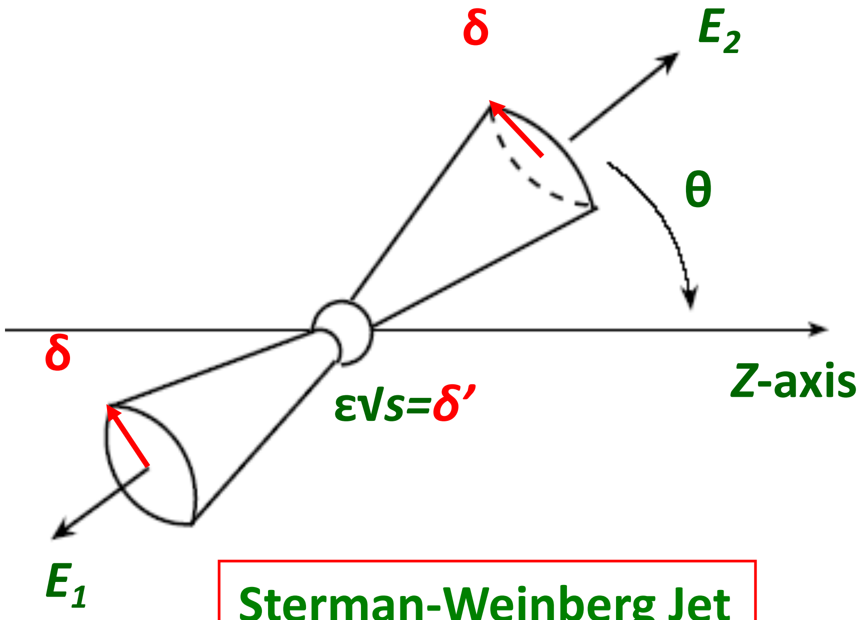
with $C_n = C_n(\delta)$

- **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as } Q\to\infty$$



Sterman-Weinberg Jet

Basics of Jet Finding Algorithms

❑ Recombination jet algorithms (almost all e+e- colliders):

Recombination metric: $y_{ij} = \dfrac{M_{ij}^2}{E_{c.m.}^2}$

$M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

for Durham k_T

- ✧ different algorithm = different choice of M_{ij}^2 :
- ✧ combine the particle pair (i,j) with the smallest y_{ij} : $(i,j) \to k$

$$e.g.\ \text{E scheme}: \ p_k = p_i + p_j$$

- ✧ iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$

❑ Cone jet algorithms (CDF,LHC, …, EIC, … colliders):

- ✧ Cluster all particles into a cone of half angle R to form a jet:
- ✧ Require a minimum visible jet energy: $E_{jet} > \epsilon$

$$\Delta_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

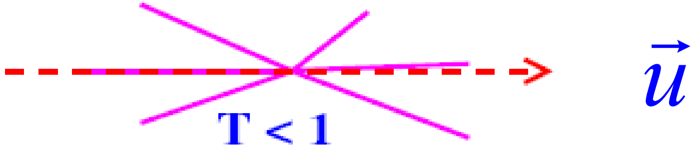
Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right)\dfrac{\Delta_{ij}^2}{R^2}$

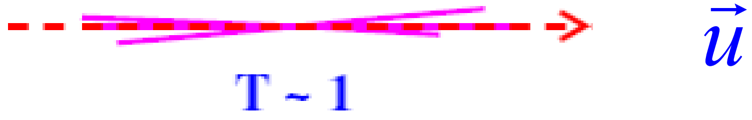
- ✧ Classical choices: *p=1* – “k_T algorithm”, *p= -1* – “anti-k_T”, …

Thrust Distribution – Event Shape

- ❑ **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^n \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^n |\vec{p}_i|} \right)$$





- ❑ **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to\text{hadrons}}}{dT} \quad \textbf{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

  - ✧ **Contribution from p=0 particles drops out the sum**
  - ✧ **Replace two collinear particles by one particle does not change the thrust:**

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

- ❑ **Other IRS event shape observables:** **Energy-energy correlations, jetness, …**

The Electron-Ion Collider (EIC)

- Lec. 1: EIC & Fundamentals of QCD
- Lec. 2: Probing Emergent Properties and Structure of Hadrons without seeing Quark/Gluon?
  – *breaking the hadron*!
- Lec. 3: Probing Structure of Hadrons without breaking them?
  – *Spin as another knob*
- Lec. 4: Dense Systems of gluons
  – *Nuclei as Femtosize Detectors*

Jianwei Qiu
Theory Center, Jefferson Lab