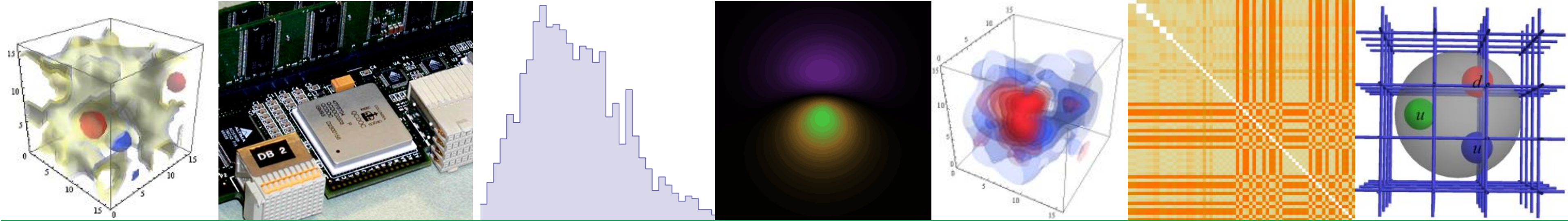


QCD on the Lattice - II

HUEY-WEN LIN

CTEQ

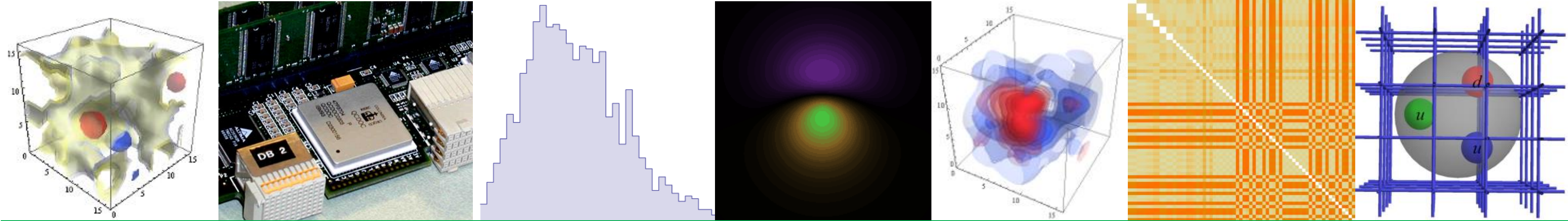
This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

NSF

U.S. DEPARTMENT OF ENERGY

@LinQCD

RESEARCH CORPORATION for SCIENCE ADVANCEMENT



Probing the Heart of Matter with Supercomputers

Huey-Wen Lin

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

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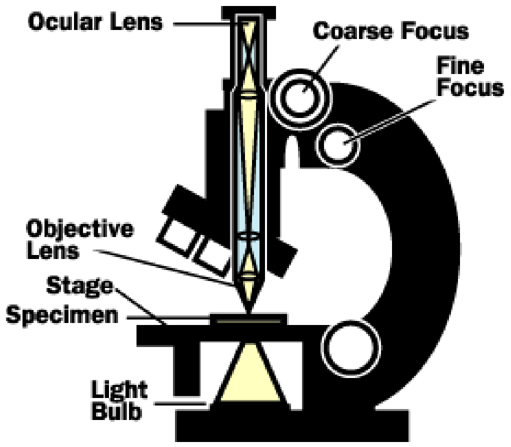
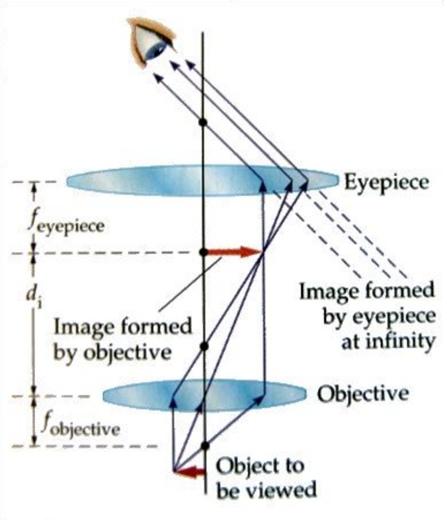
RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Topic in QCD: Structure

- What is the structure of the nucleon?

probing insights into nucleons





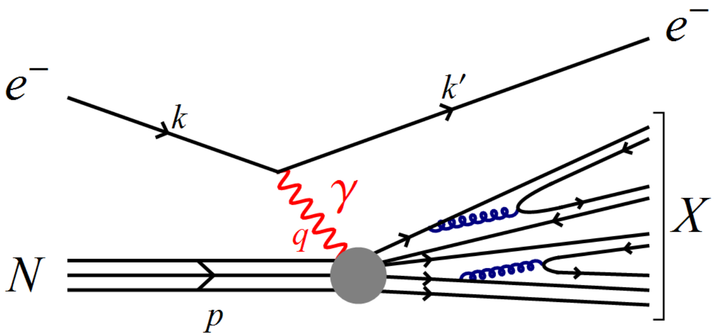
Topic in QCD: Structure

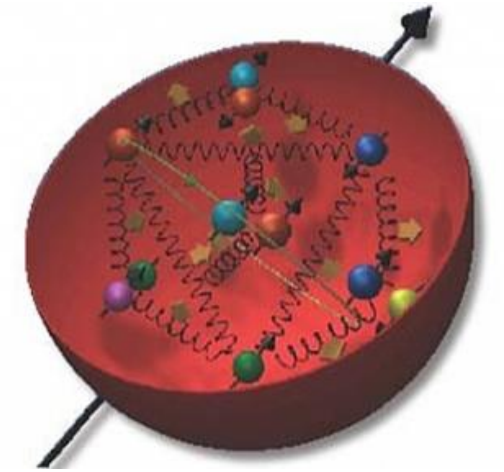
- What is the structure of the nucleon?

  probing insights into nucleons

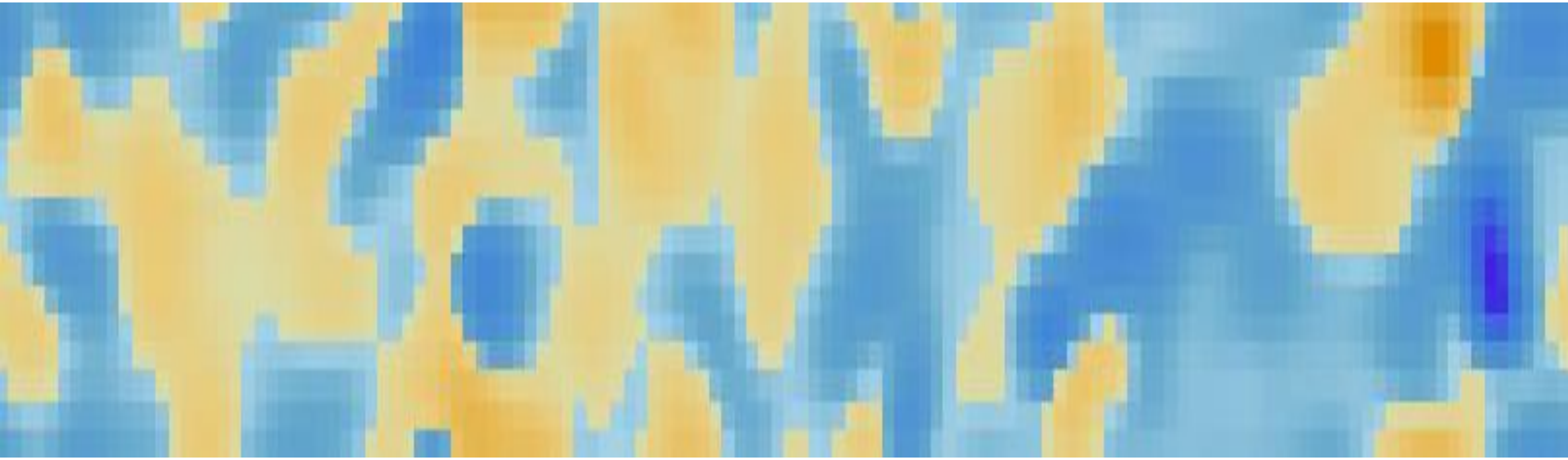
BNL

JLAB





Nucleon Matrix Elements

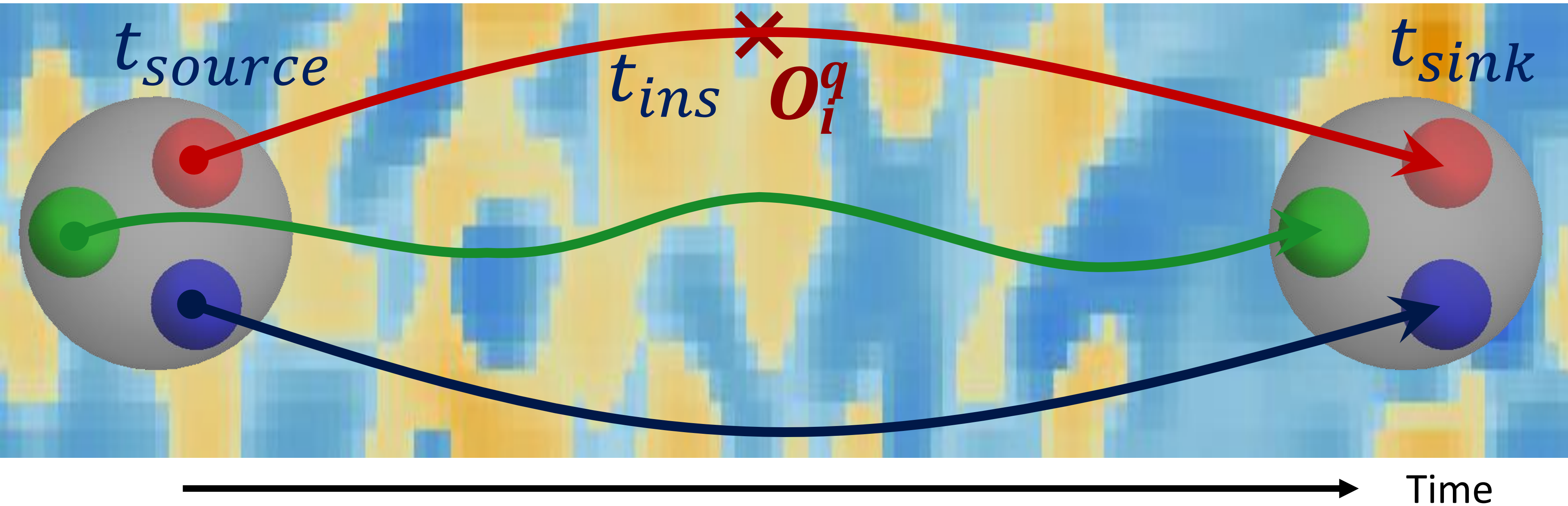


§ Pick a QCD vacuum

- Gauge/fermion actions, flavor (2, 2+1, 2+1+1), m_π, a, L, ...

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

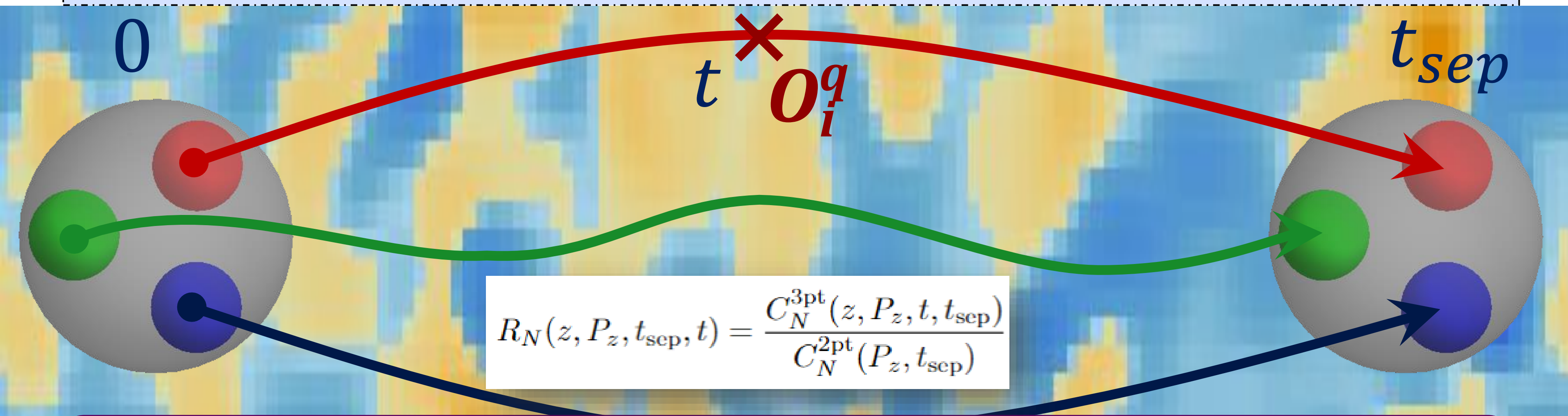


§ Construct correlators (hadronic observables)

- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Exercise: 3) Derive the time-dependent formula for the three-point correlator. Start with the pion case.



$$R_N(z, P_z, t_{\rm sep}, t) = \frac{C_N^{\rm 3pt}(z, P_z, t, t_{\rm sep})}{C_N^{\rm 2pt}(P_z, t_{\rm sep})}$$

- § Careful analysis needed to remove systematics
  - Wrong results if **excited-state systematic** is not under control

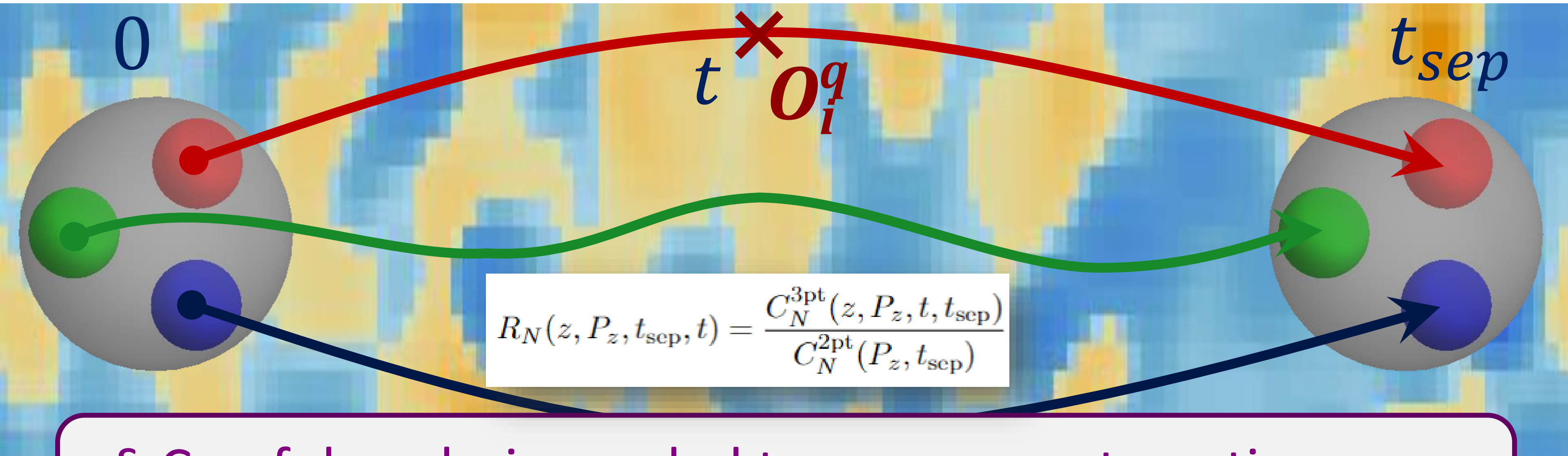
$$C_N^{\rm 3pt}(z, P_z, t, t_{\rm sep}) \propto |A_{N,0}|^2 \langle 0|O_g|0\rangle e^{-E_{N,0}t_{\rm sep}} + |A_{N,0}||A_{N,1}|\langle 0|O_g|1\rangle e^{-E_{N,1}(t_{\rm sep}-t)}e^{-E_{N,0}t}$$

$$+ |A_{N,0}||A_{N,1}|\langle 1|O_g|0\rangle e^{-E_{N,0}(t_{\rm sep}-t)}e^{-E_{N,1}t} + |A_{N,1}|^2 \langle 1|O_g|1\rangle e^{-E_{N,1}t_{\rm sep}}$$

$$C_N^{\rm 2pt}(P_z, t) \propto |A_{N,0}|^2 e^{-E_{N,0}t} + |A_{N,1}|^2 e^{-E_{N,1}t} + \ldots,$$

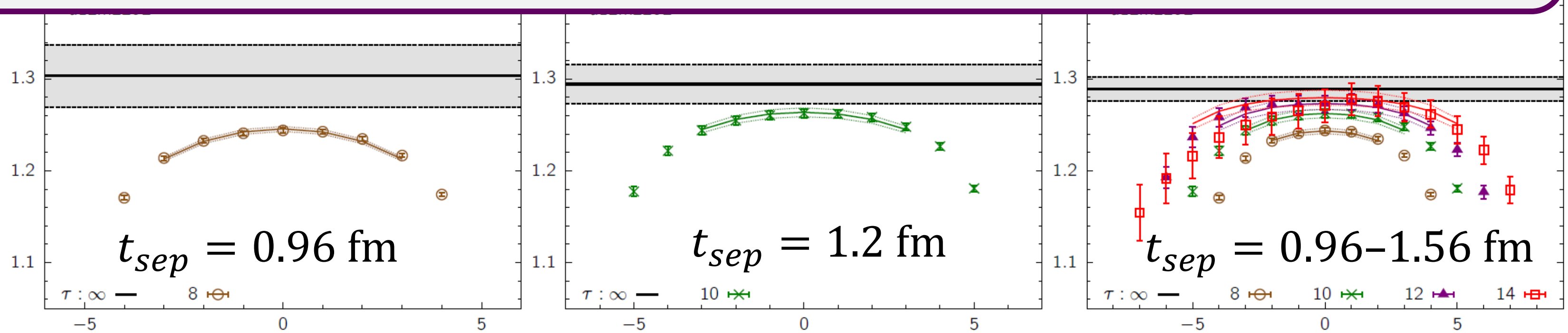
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



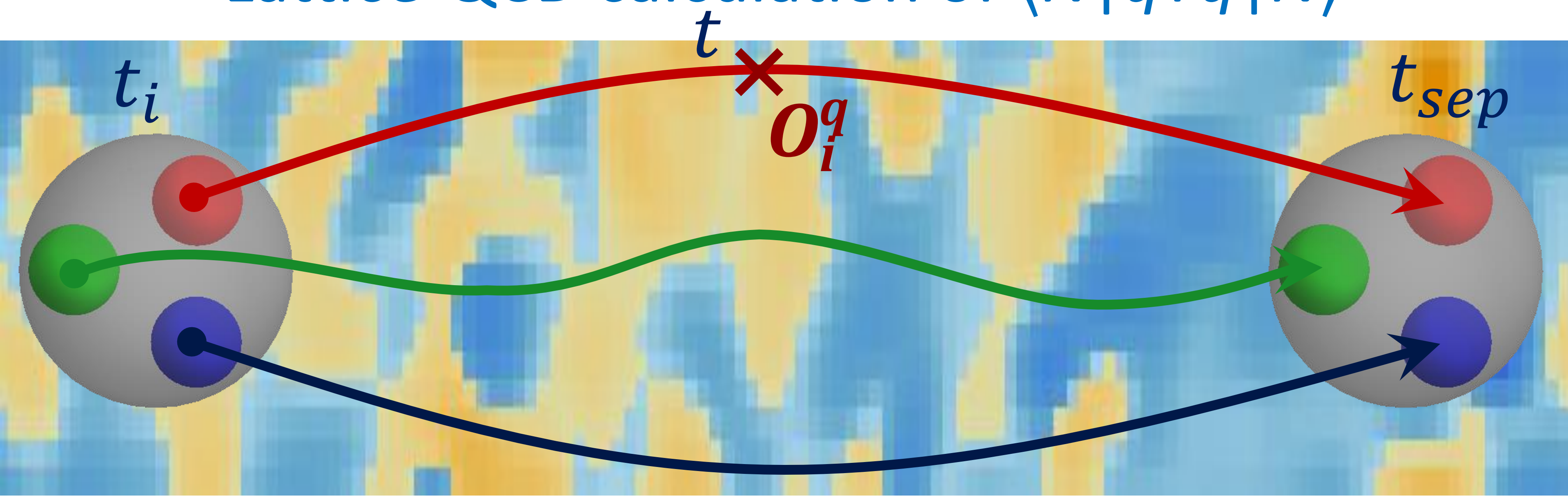
- § Careful analysis needed to remove systematics
  - Wrong results if **excited-state systematic** is not under control

Ratio Plot



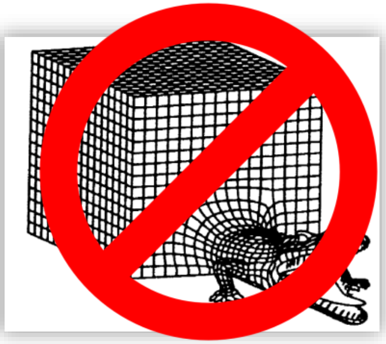
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Systematic uncertainty (nonzero a, finite L, etc.)

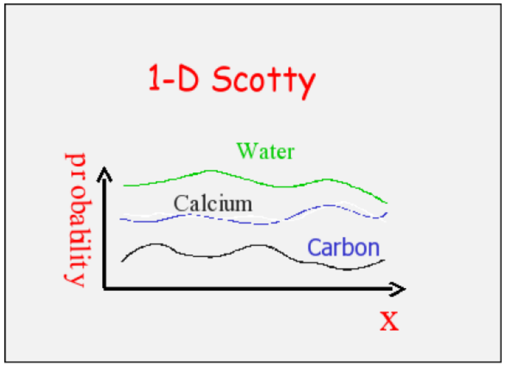
- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Hadron Structure

§ Structure function/distribution functions

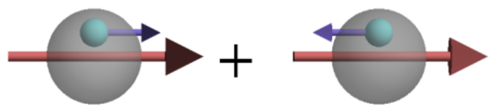
- deep inelastic scattering
- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$



$$\mathcal{O}^{\mu\nu}_{V^a} = \overline{q}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\tau^a q\,,$$
$$\mathcal{O}^{\mu\nu}_{A^a} = \overline{q}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\gamma^5\tau^a q\,,$$
$$\mathcal{O}^{\mu\nu\rho}_{T^a} = \overline{q}\sigma^{[\mu\{\nu]}\overleftrightarrow{D}^{\rho\}}\tau^a q\,,$$

Moments of PDFs

§ First moments are most commonly done

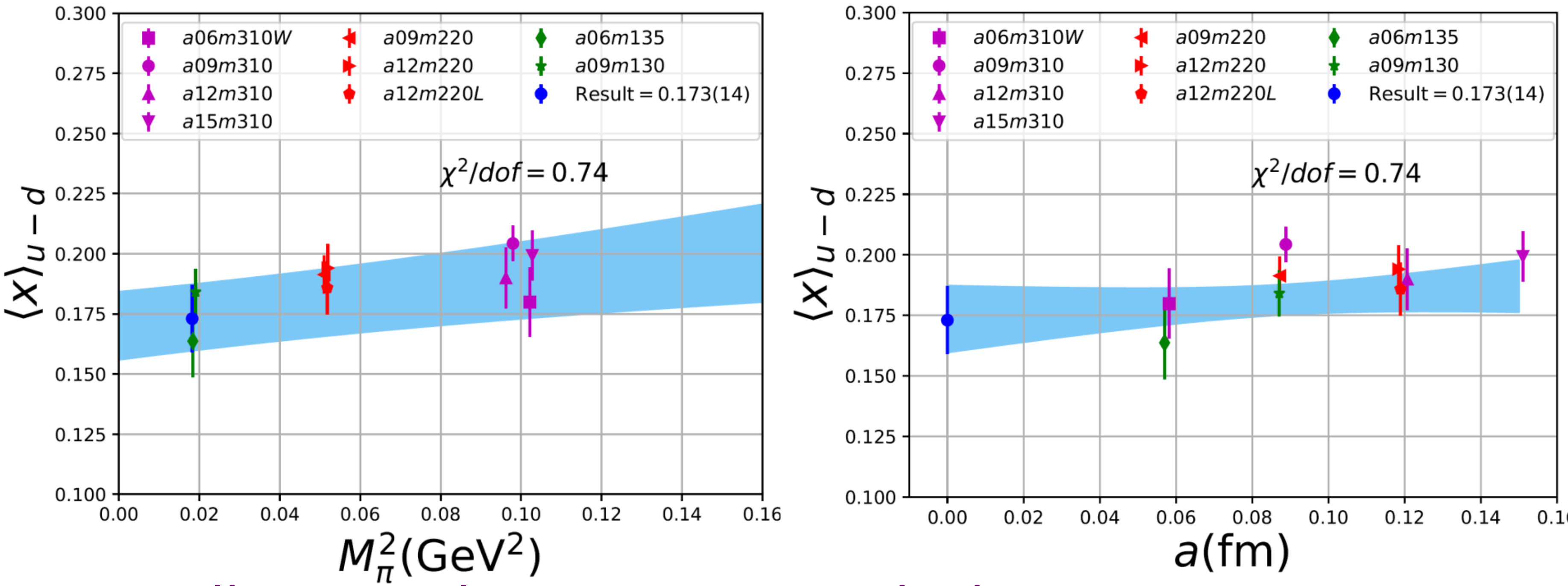


§ State-of-the art example

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

- Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779



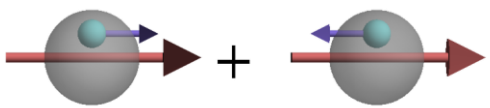
§ Usually more than one LQCD calculation

- Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

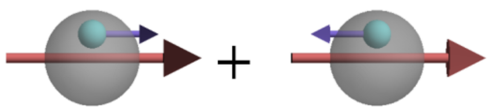
Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		0.180(25)($^{+14}_{-6}$)	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

** No quenching effects are seen.

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



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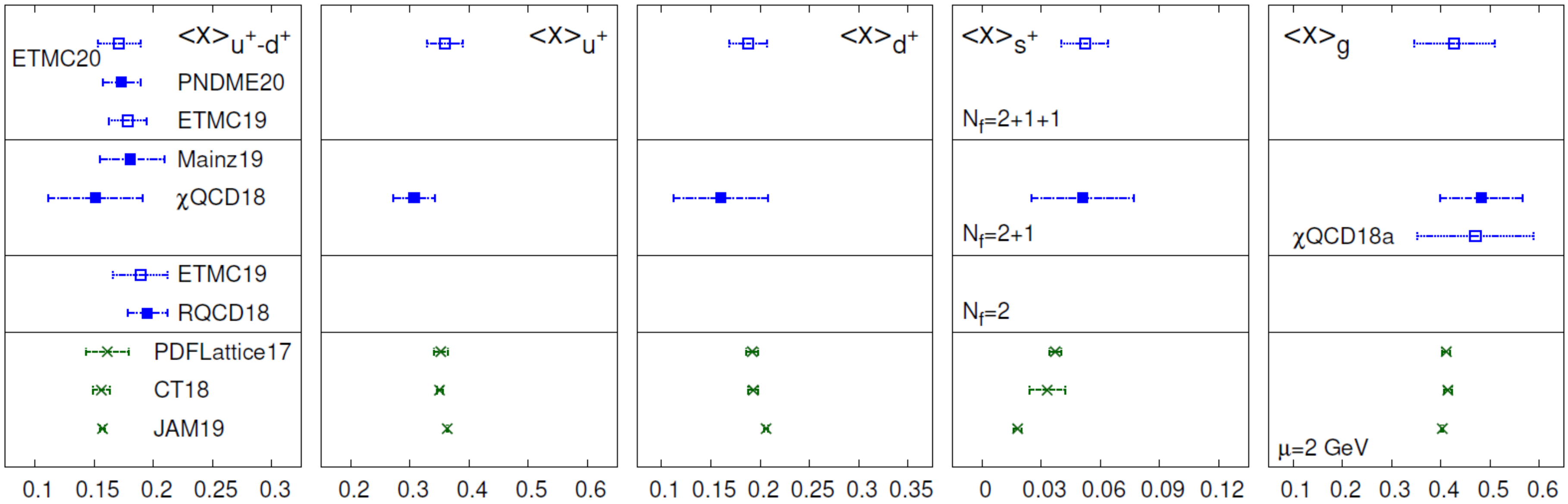
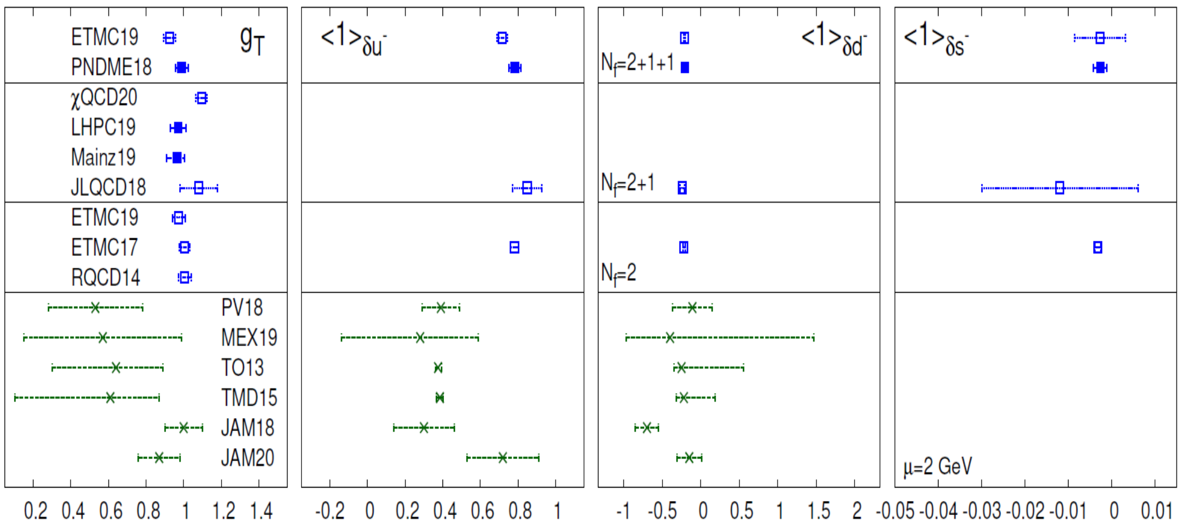


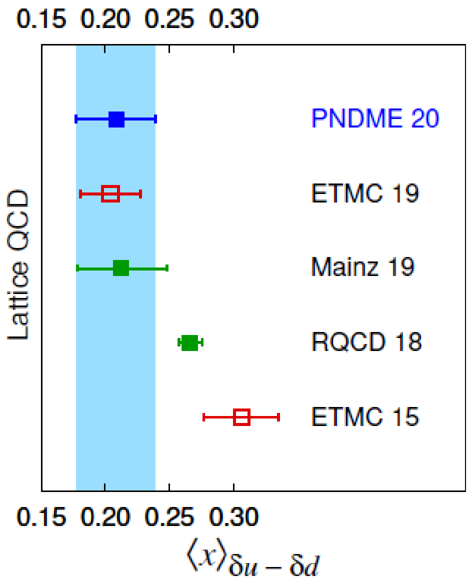
Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636



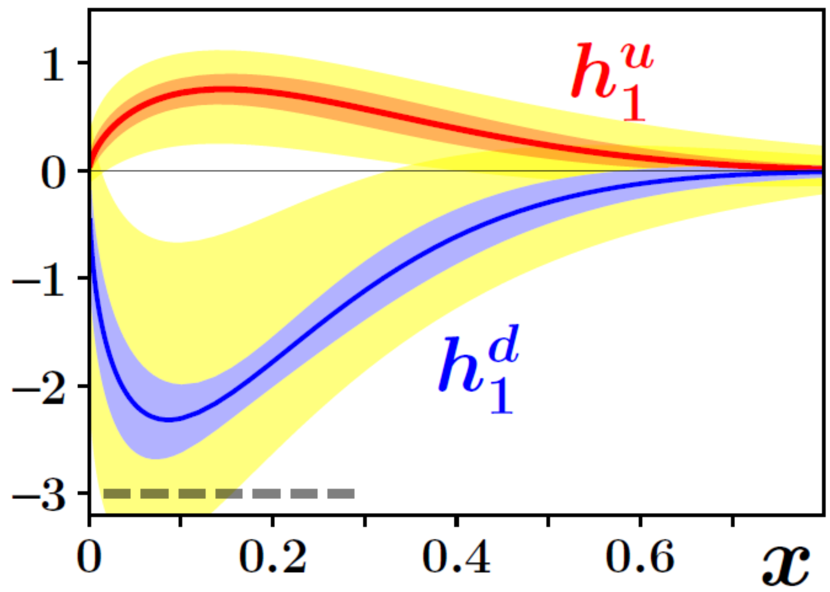


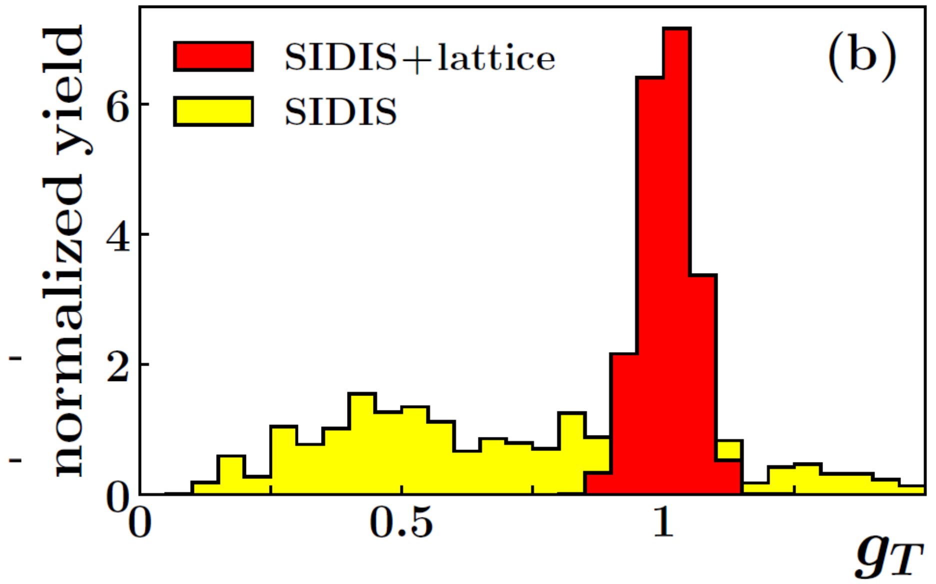
S. Mondal et al (PNDME), 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets





Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

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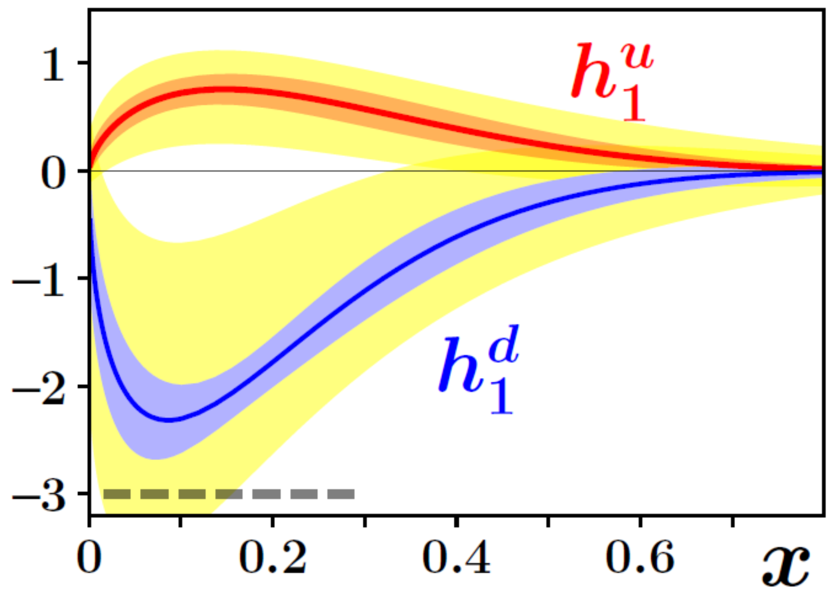
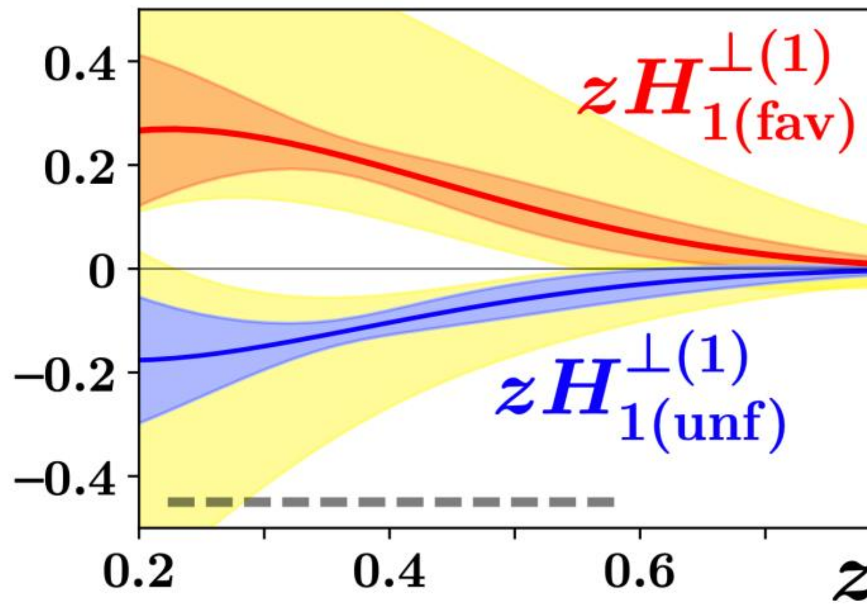


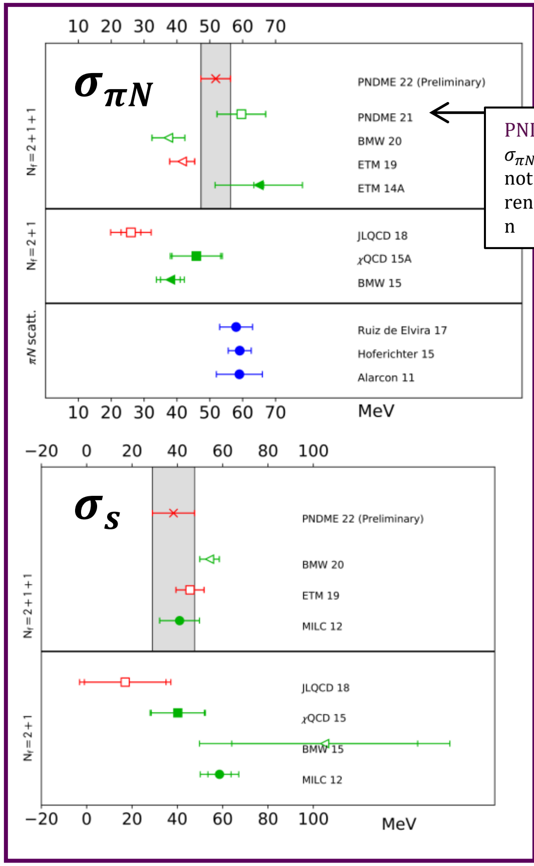


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Nucleon Flavor Diagonal Charges

Comparison with FLAG 2021 results

Nucleon sigma terms (Scalar charges)

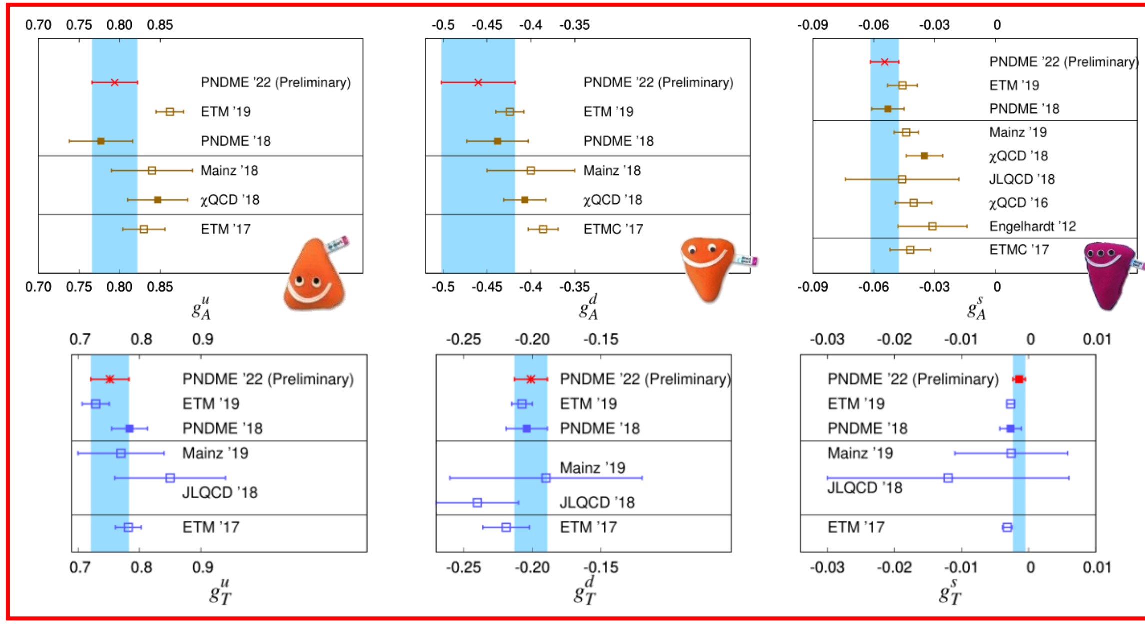


PNDME (2021) $\sigma_{\pi N}$ which does not require renormalizatio n

[PNDME collab., Lattice 2022 update, **preliminary**]

- Clover fermion on $N_f = 2 + 1 + 1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- **Chiral-Continuum extrapolation** including a data at M_π^{Phys}

Axial and Tensor charges



Plots by Sungwoo Park

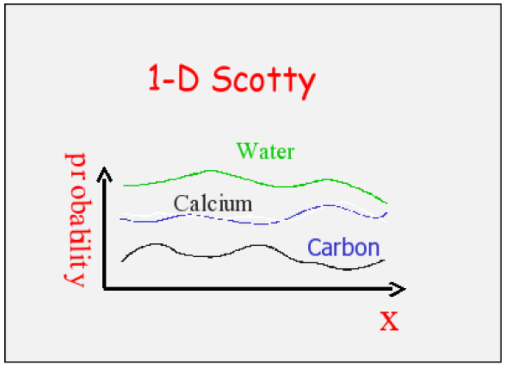
Hadron Structure

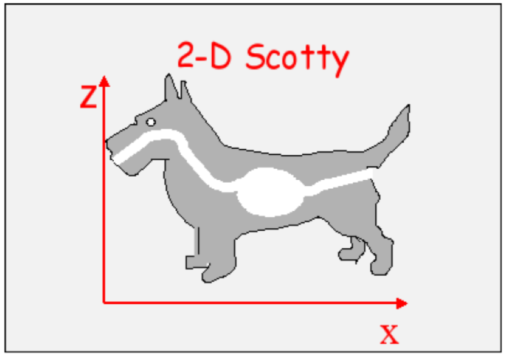
§ Structure function/distribution functions

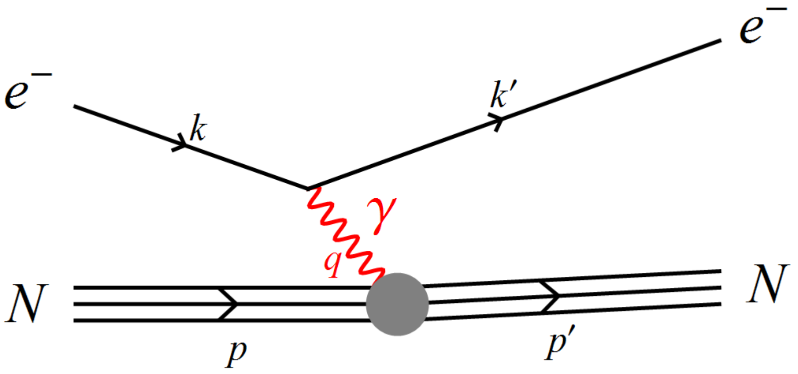
- deep inelastic scattering
- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$

§ Form factors

- elastic scattering
- $F_1(Q^2)$, $F_2(Q^2)$, $G_A(Q^2)$, $G_P(Q^2)$



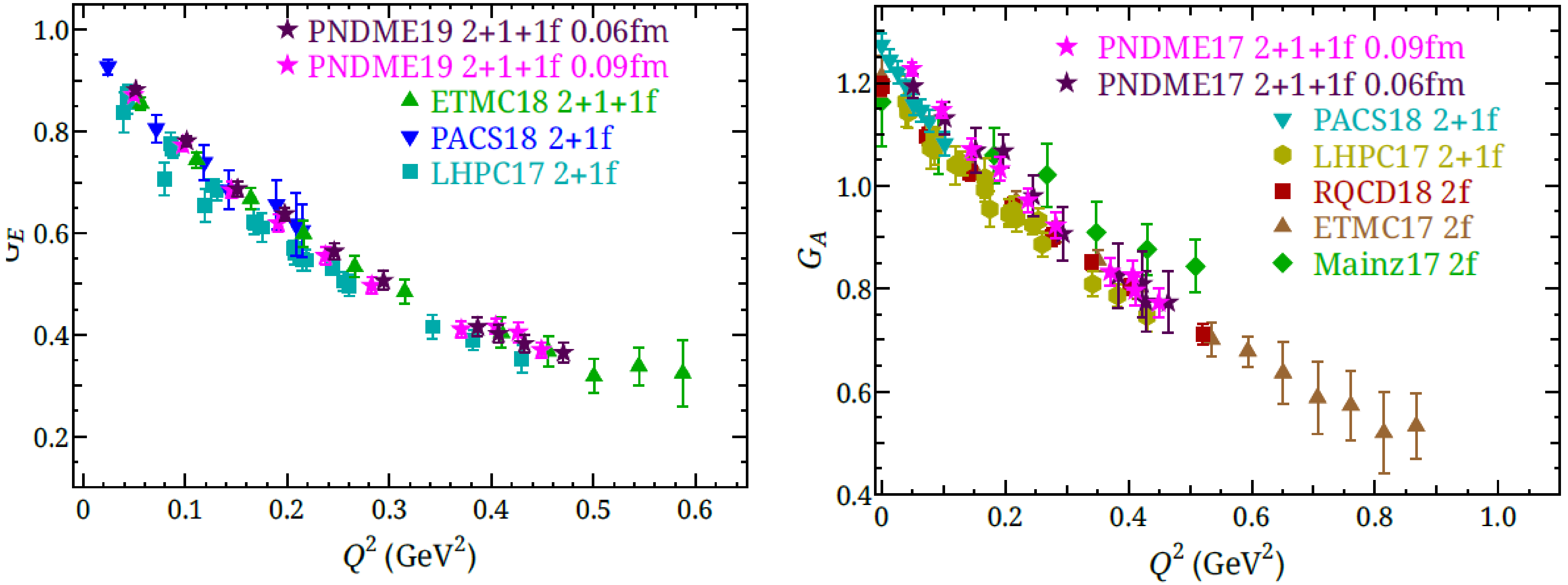




Nucleon Form Factors

§ Nucleon *isovector* electromagnetic and axial form factor

- Many existing LQCD works in the past few decades
- Worldwide:
  Increasingly many ensembles available at physical pion mass

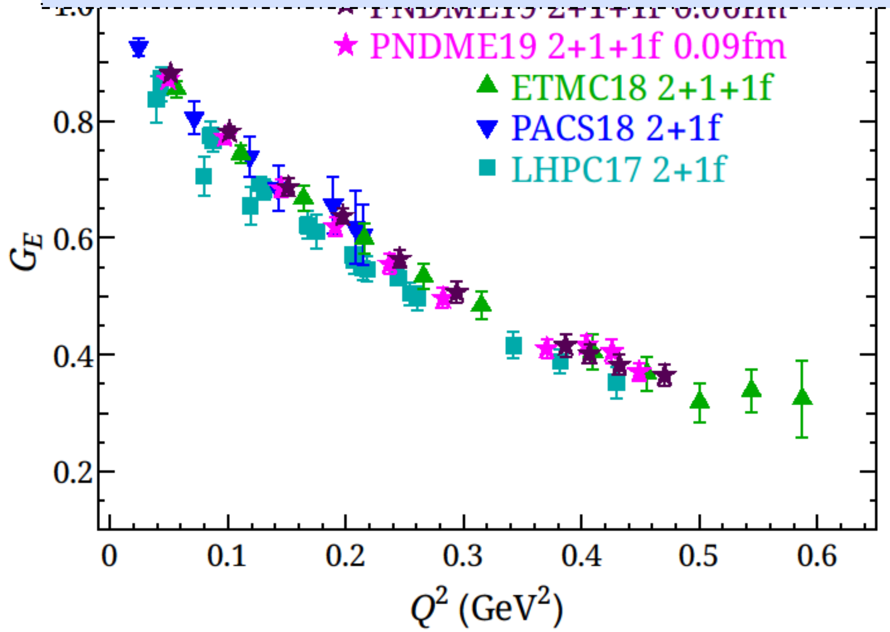


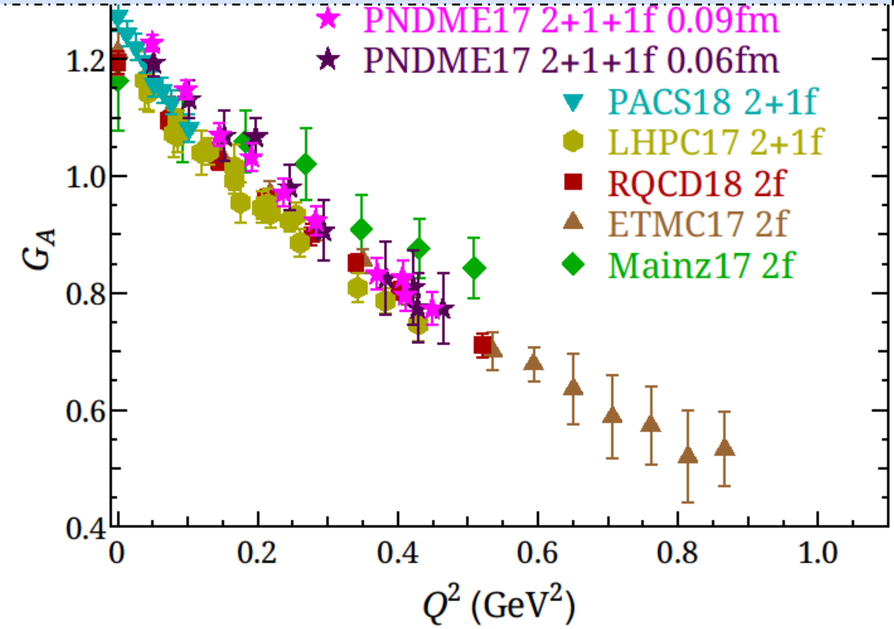
HL, Int. J. Mod. Phys. A 35 (2020) 11n12, 2030006

Form Factors

§ Nucleon *isovector* electromagnetic and axial form factor

Exercise: 4) Given what you learned about the time dependence of the 3pt correlator, what do you expect to see on the ratio plots of the form-factor data?





$$\langle N(\boldsymbol{p}_f)|V_\mu(\boldsymbol{q})|N(\boldsymbol{p}_i)\rangle = \overline{u}_N(\boldsymbol{p}_f)\left(F_1(Q^2)\gamma_\mu + \sigma_{\mu\nu}\frac{F_2(Q^2)}{2M_N}\right)u_N(\boldsymbol{p}_i)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2}F_2(Q^2)$$

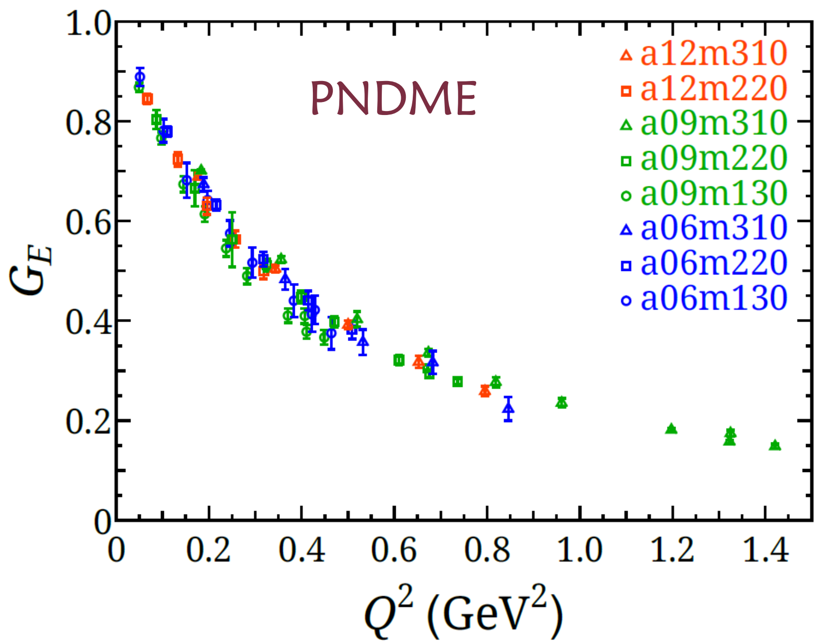
$$\langle N(\boldsymbol{p}_f)|A_\mu(\boldsymbol{q})|N(\boldsymbol{p}_i)\rangle = \overline{u}_N(\boldsymbol{p}_f)\left(G_A(Q^2)\gamma_\mu + q_\mu\frac{\tilde{G}_P(Q^2)}{2M_N}\right)\gamma_5 u_N(\boldsymbol{p}_i)$$

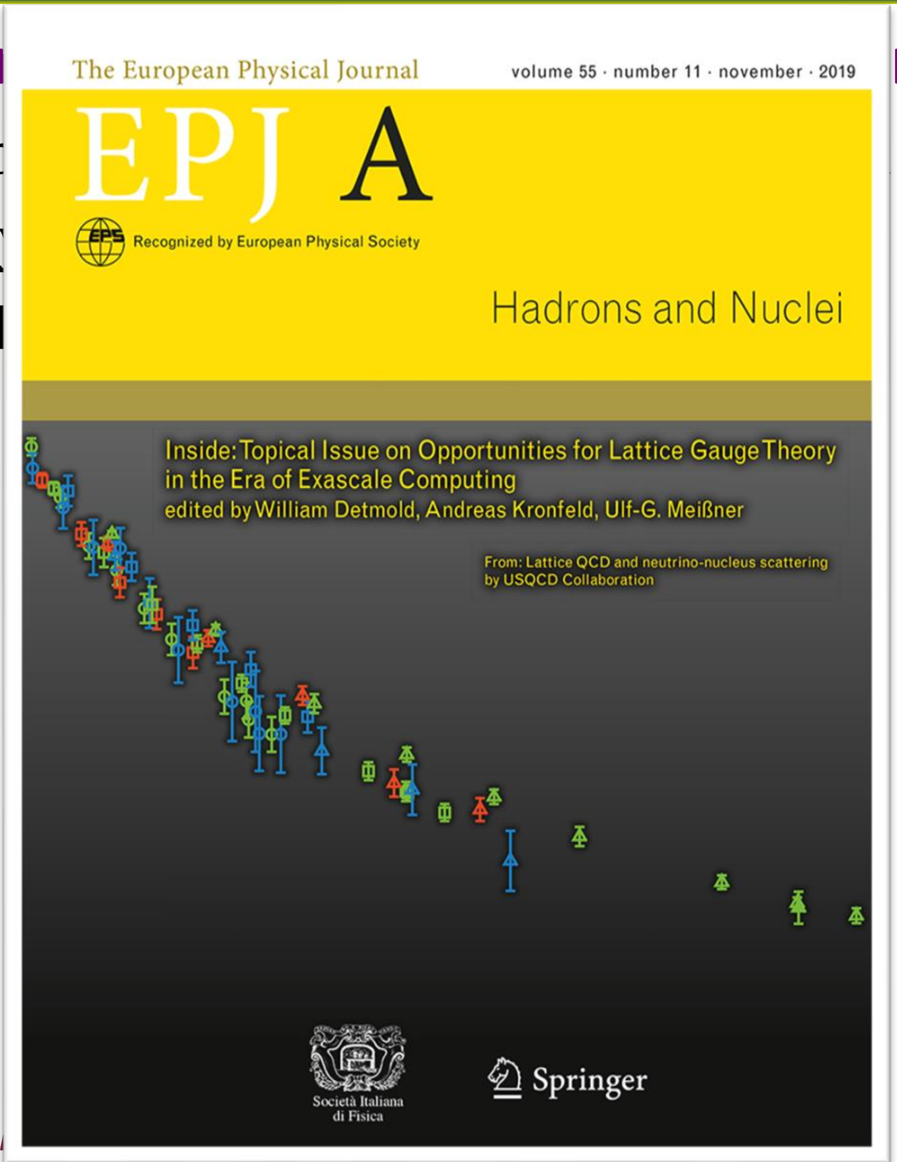
HL, Int. J. Mod. Phys. A 35 (2020) 11n12, 2030006

Nucleon Form Factors

§ Nucleon *isovector* electro

- Name of the game is now "p
- Example work supported by (Studying multiple lattice sp control systematics)



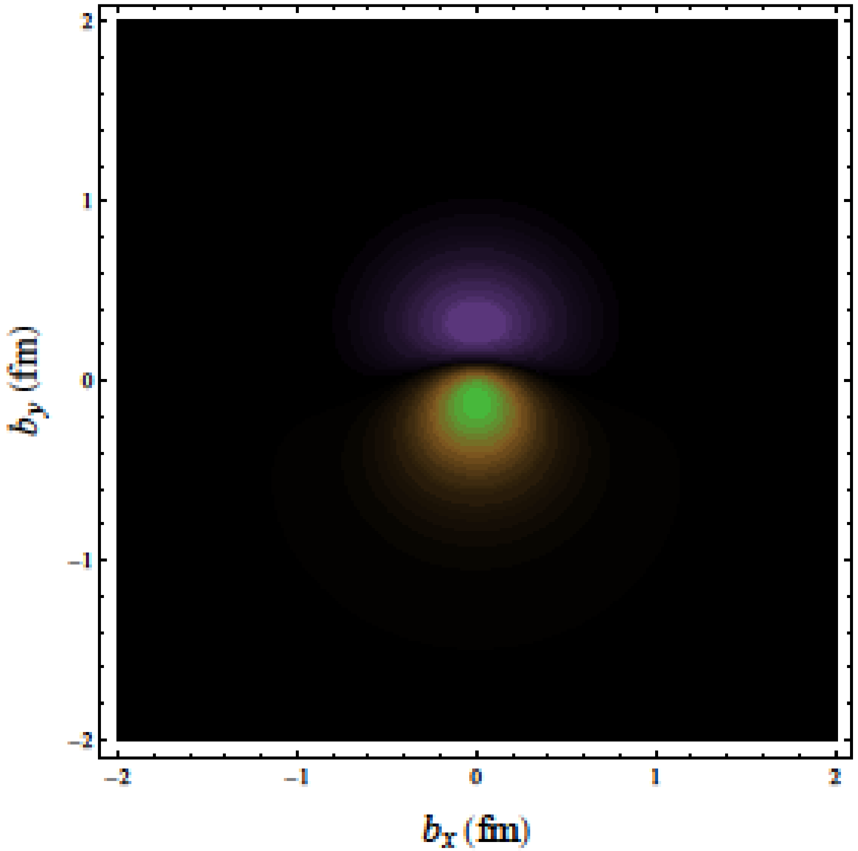


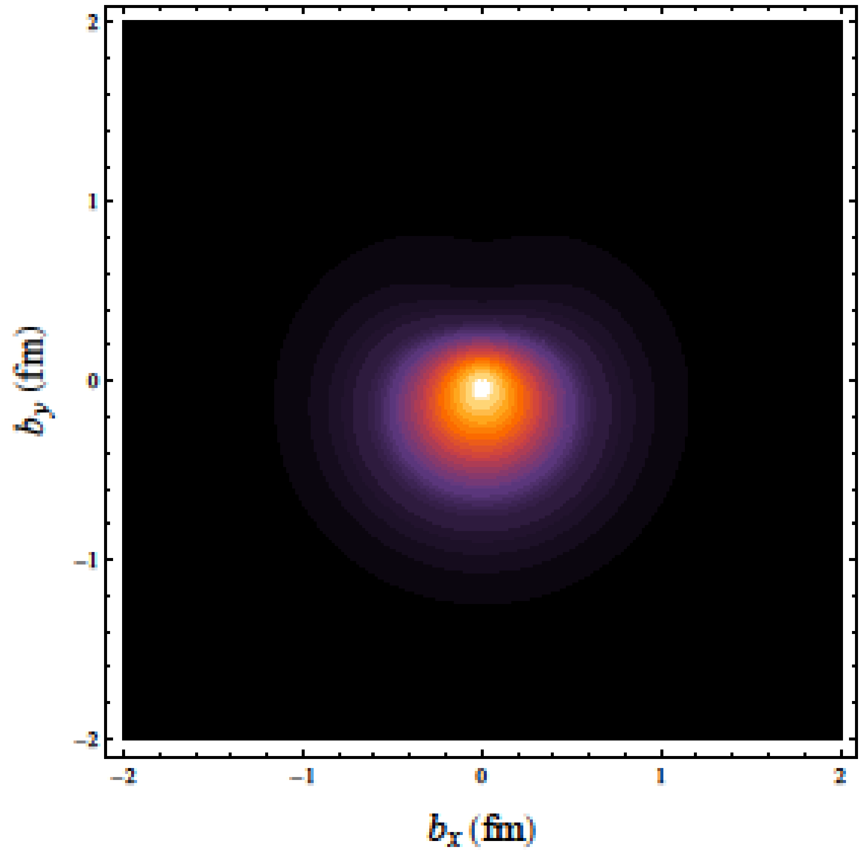
USQCD Collaboration, Eur.Phys.J.A

Mapping Nucleon Picture

- Fourier transform using large-Q^2 form factors to reveal transverse charge densities in a polarized nucleon

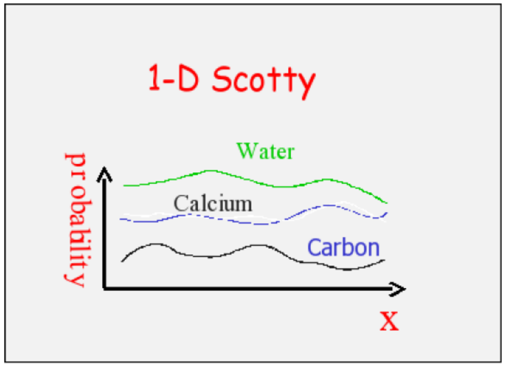
HWL, National Academies Press

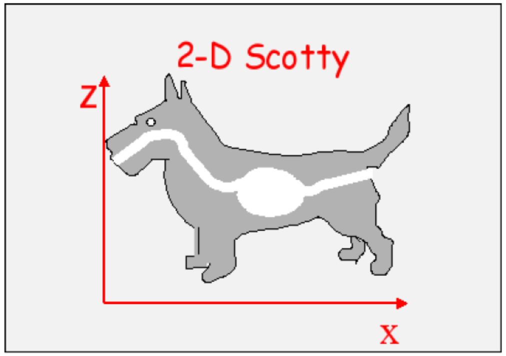


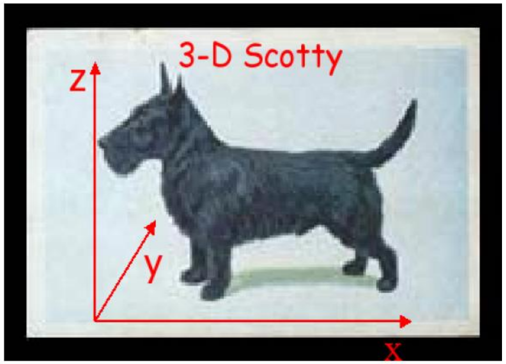


Hadron Structure

- § Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n\rangle_q$, $\langle x^n\rangle_{\Delta q}$, $\langle x^n\rangle_{\delta q}$
- § Form factors
  - elastic scattering
  - $F_1(Q^2)$, $F_2(Q^2)$, $G_A(Q^2)$, $G_P(Q^2)$
- § Generalized Parton Distribution
  - DVCS
  - $\langle x^{n-1}\rangle_q = A_{n0}(0)$, $\langle x^{n-1}\rangle_{\Delta q} = \tilde{A}_{n0}(0)$, $\langle x^n\rangle_{\delta q} = A_{Tn0}(0)$,
  - $F_1(Q^2) = A_{10}(Q^2)$, $F_2(Q^2) = B_{20}(Q^2)$, $G_A(Q^2) = \tilde{A}_{10}(Q^2)$, $G_P(Q^2) = \tilde{B}_{10}(Q^2)$
  - Nucleon spin



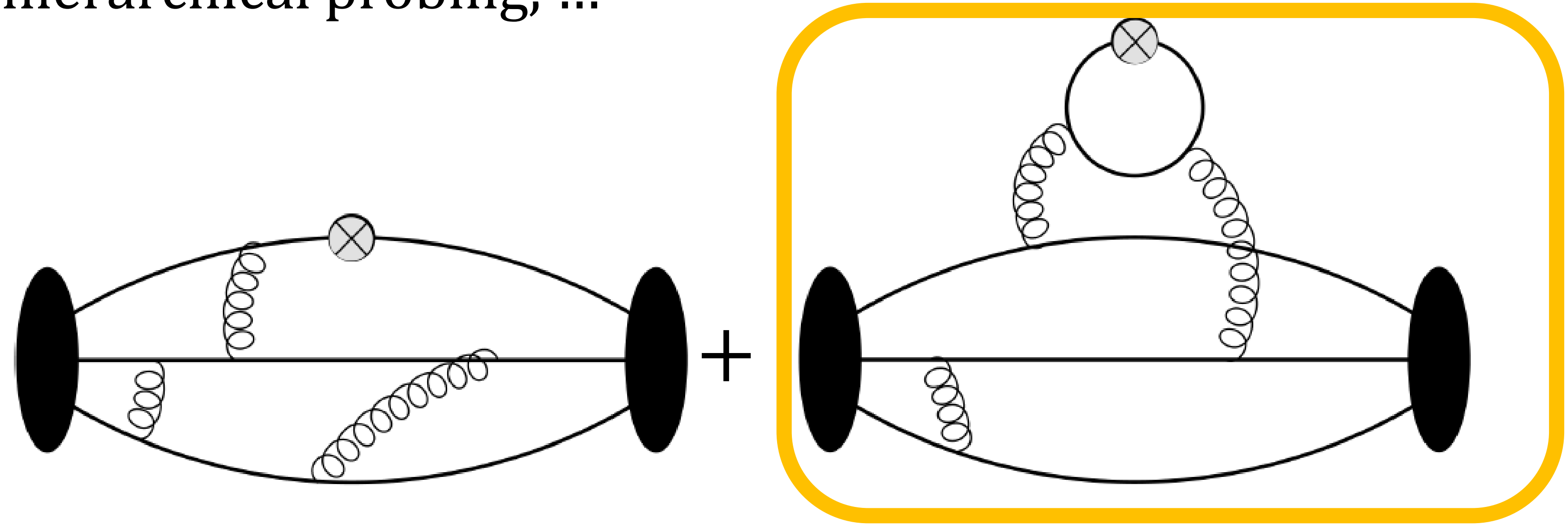




Disconnected Diagrams

§ Disconnected diagram

- Multiple ways to calculate this notorious contribution…
- Truncated solver, hopping-parameter expansion, hierarchical probing, …



Spin Decomposition

§ Orbital angular momentum from Ji definition

$$\vec{J}_g = \int d^3x \, [\vec{x} \times (\vec{E}\times\vec{B})]$$

$$\vec{J}_q = \int d^3x \, \psi^\dagger[\vec{\gamma}\gamma_5 + \vec{x}\times(-i\vec{D})]\psi = \frac{1}{2}(A_{20}^q + B_{20}^q)$$

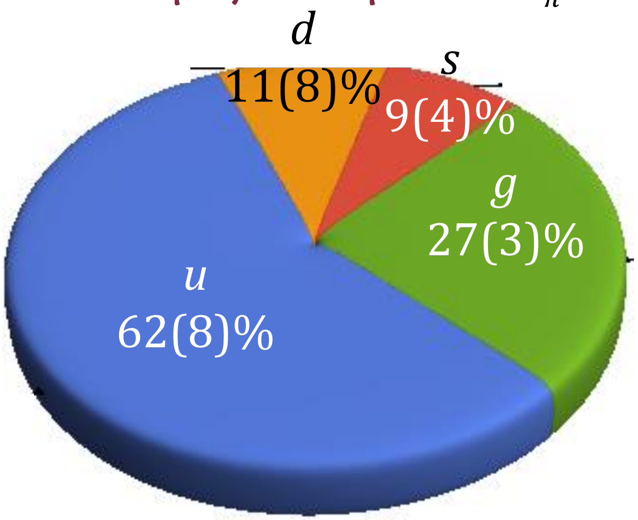
quark spin

quark orbital angular momentum

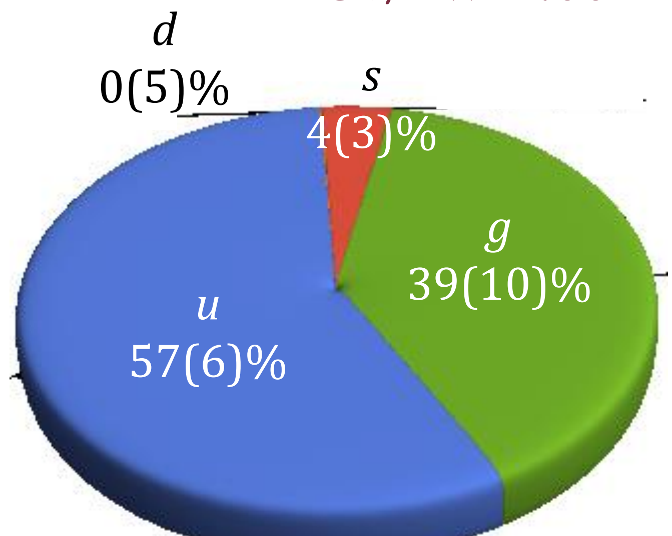
obtained using GPD moment

§ Total quark and gluon contributions

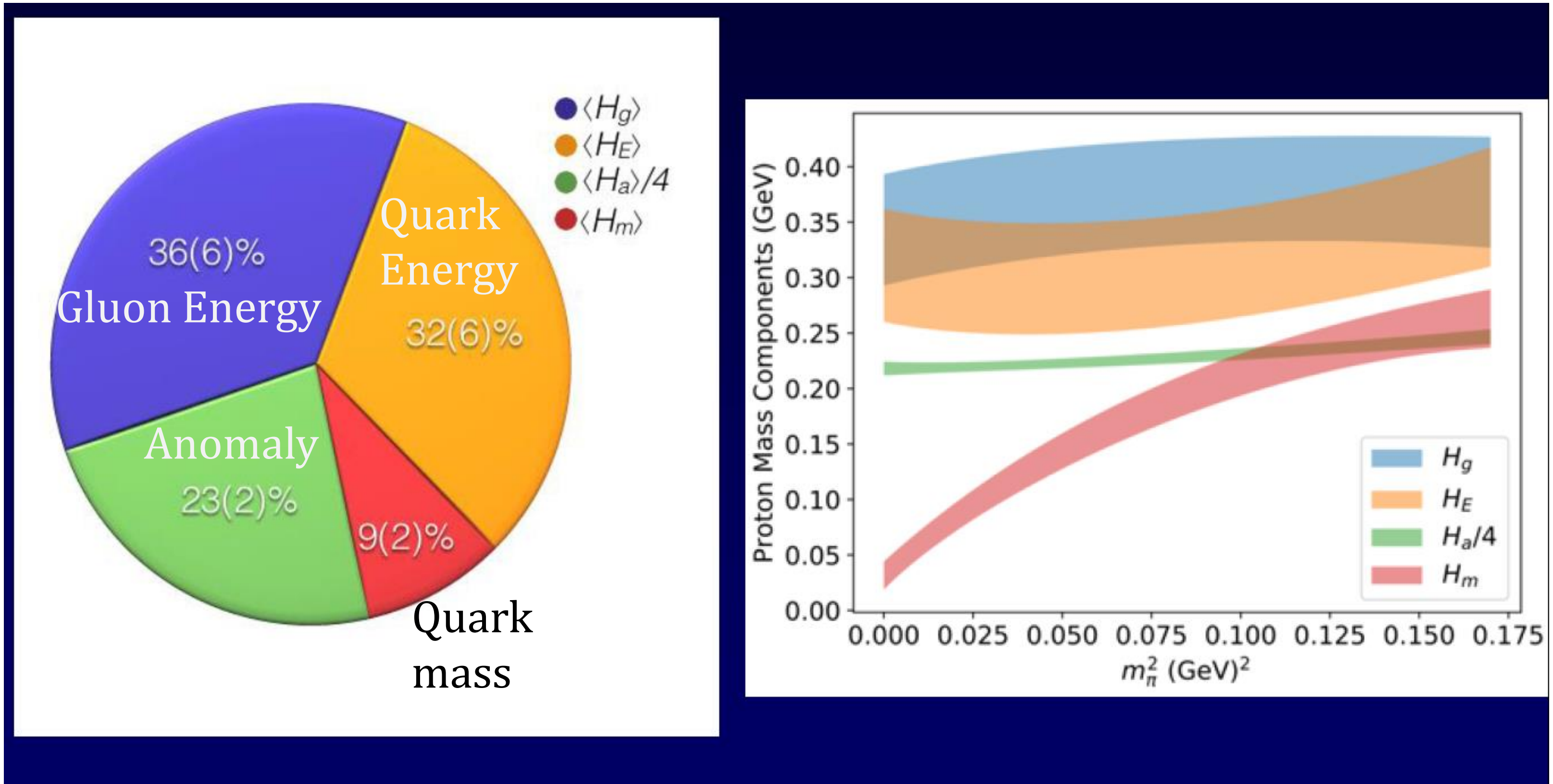
ETMC, 1706.02973
2f TM+clover, physical quark, $M_\pi L < 3$



χQCD, 1904.04138
2+1f Ov/DWF 400 MeV



Proton Mass Decomposition



Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)

Slide from Keh-Fei Liu

Nucleons and New Physics

Many opportunities to probe new physics with nucleon inputs

- § Parton distribution functions for SM background
  - Especially less known intrinsic strange/charm contribution
- § Dark matter detection Phys.Rev.D 89 (2014) 074505; ongoing work
  - Popular candidates (e.g. SuSy neutralinos) exchange Higgs
- § Electric dipole moment Phys.Rev.Lett. 115 (2015) 21, 212002 (108 citations); Phys.Rev.D 98 (2018) 9, 091501 (43 citations)
  - CP-violating effect , extremely small: in SM $\approx 10^{-30}$ e-cm
- § Neutron beta decay PNDME, Phys.Rev.D 98 (2018) 034503 (84 citations); Phys.Rev.D 94 (2016) 5, 054508 (120 citations)
  - Non-V–A interactions to probe the existence of new particles (mediating new forces) with masses in the multi-TeV range
- § Nucleon (transition) form factors Phys. Rev. D 96, 114503 (67 citations)
  - First-principles inputs for precision neutrino physics

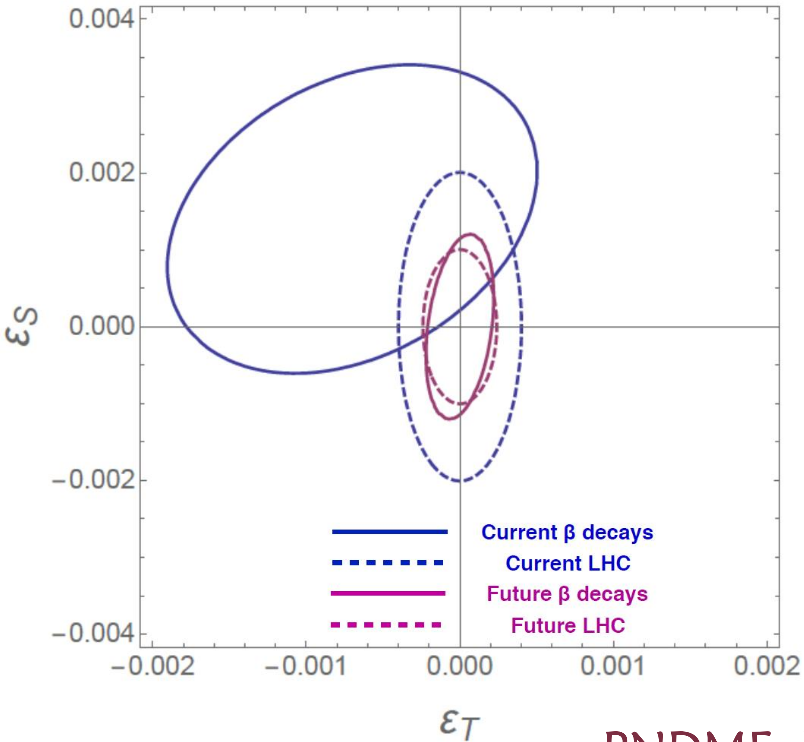
Many of these are supported by P5 recommendations or ongoing Snowmass efforts

Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Low-Energy Expt → $O_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)

$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$



Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$|B_1 - b|_{\text{BSM}} < 10^{-3}$

$|b|_{\text{BSM}} < 10^{-3}$

CENPA: $^6\text{He}(b_{\text{GT}})$ at 10^{-3}

Plots by Vincenzo Cirigliano

PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049; 1806.09006

Electric Dipole Moment

§ Why do we care?

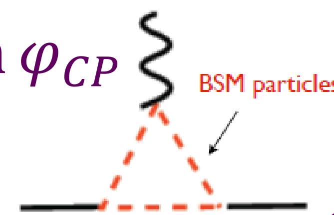
- CP-violating effect ⇒ Key ingredient for baryogenesis
  ⇒ Why matter exists
- Extremely small in SM: ≈ 10^{-31} e-cm (expect to probe 10^{-28} soon)
- Good candidate to constrain BSM models

§ Lagrangian

$$L = L_{\mathrm{QCD}}^{CP\ \mathrm{Even}} + L_{\Theta} + L_{\mathrm{quark}}^{\mathrm{dim-5}} + L_{\mathrm{chromo-quark}}^{\mathrm{dim-5}} + \cdots$$

Induced by a variety of BSM scenarios

$$d_i \propto \frac{m_i}{\Lambda^2} \sin \varphi_{CP}$$



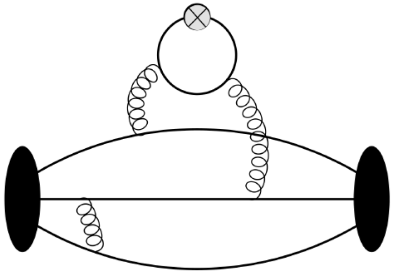
- If experiment sees signal before SM background
  ⇒ new physics
  ⇒ quark EDM (our focus here)

§ Lattice community are working on various contributions

Electric Dipole Moment

§ Quark EDM (d_q) in nucleon comes from

$O = \bar{q}\sigma_{\mu\nu}q$



$$d_N = d_u g_T^{(n,u)} + d_d g_T^{(n,d)} + d_s g_T^{(n,s)}$$

§ Extrapolate to the continuum limit

PNDME, 1806.09006, 1808.07597

$$g_T^u = 0.784(28),\ g_T^d = -204(11),\ g_T^s = -0.0027(16)$$

§ Implications for new physics?

Wells, 2003;
Arkani-Hamed and Dimopoulos, 2004;
Giudice and Romanino, 2004

- Take split SUSY for example
- Using our lattice inputs, we can derive an upper limit for the neutron EDM in split SUSY

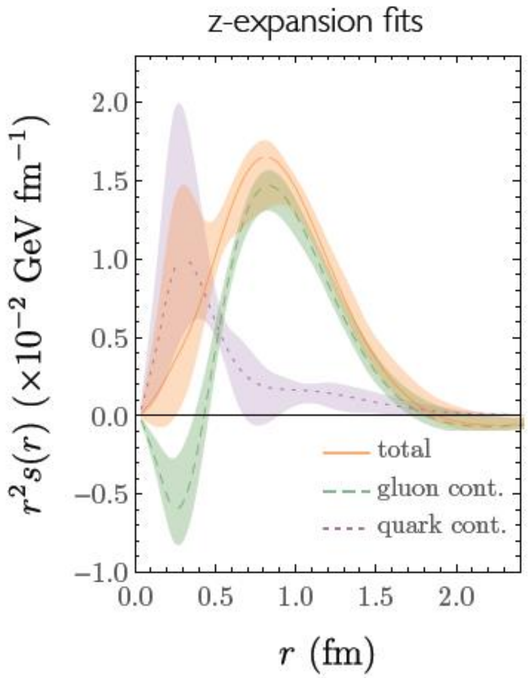
$$|d_n| < 4 \times 10^{-28} e \cdot \mathrm{cm}$$

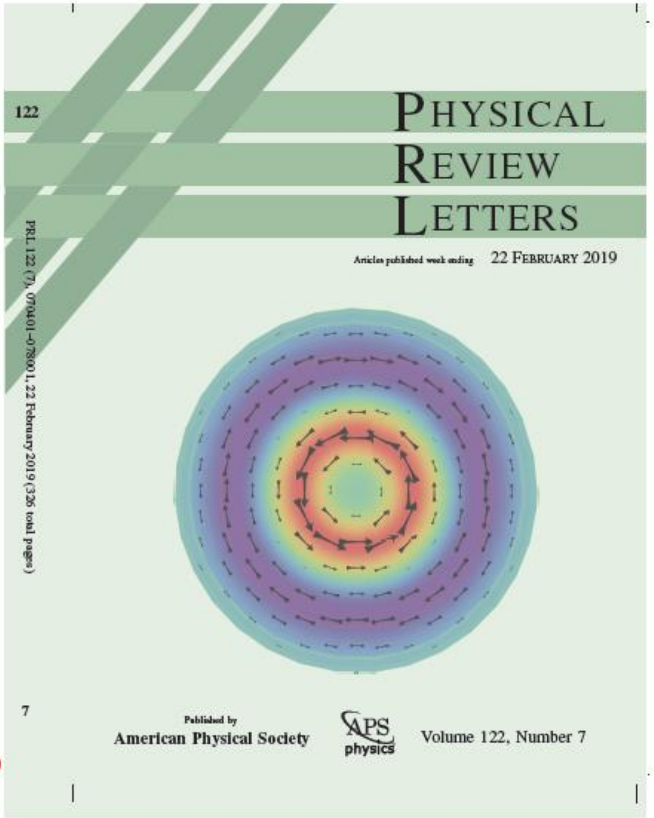
using $|d_e| < 8.7 \times 10^{-29} e \cdot \mathrm{cm}$ with 90% confidence

ACME Coll., Science Vol. 343 no. 6168 pp. 269-272 (2014)
PNDME, 1806.09006, 1808.07597

Gluon GFF

LQCD proton shear





Gluon GFFs: Shanahan, Detmold, PRD 99, 014511, PRL 122 072003 (20
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Slide by Phiala Shanahan @ Lattice PDF workshop

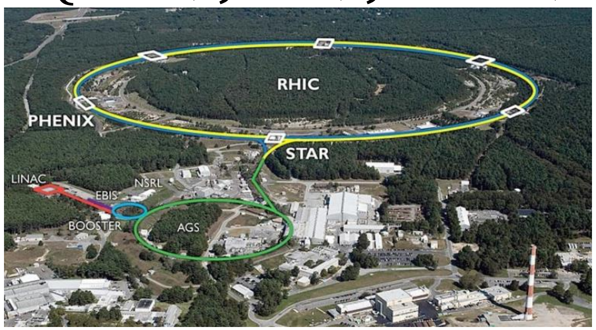
Recent Lattice PDFs Progress

Biased selected/highlighted results

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

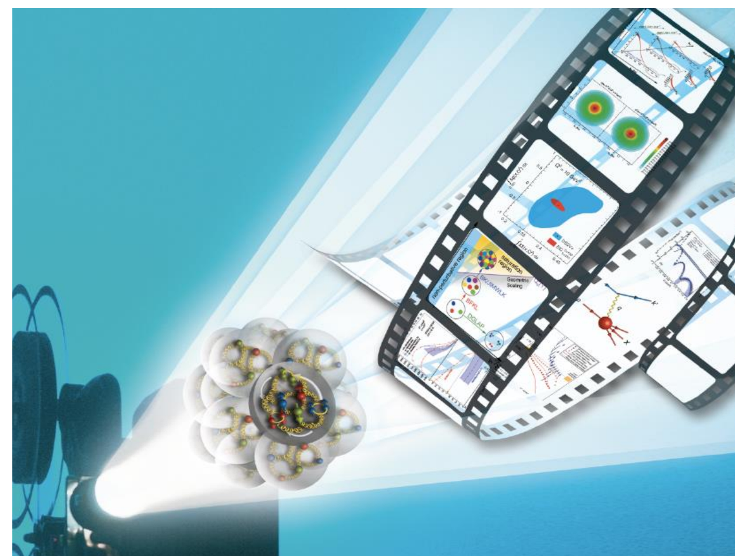
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)



Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

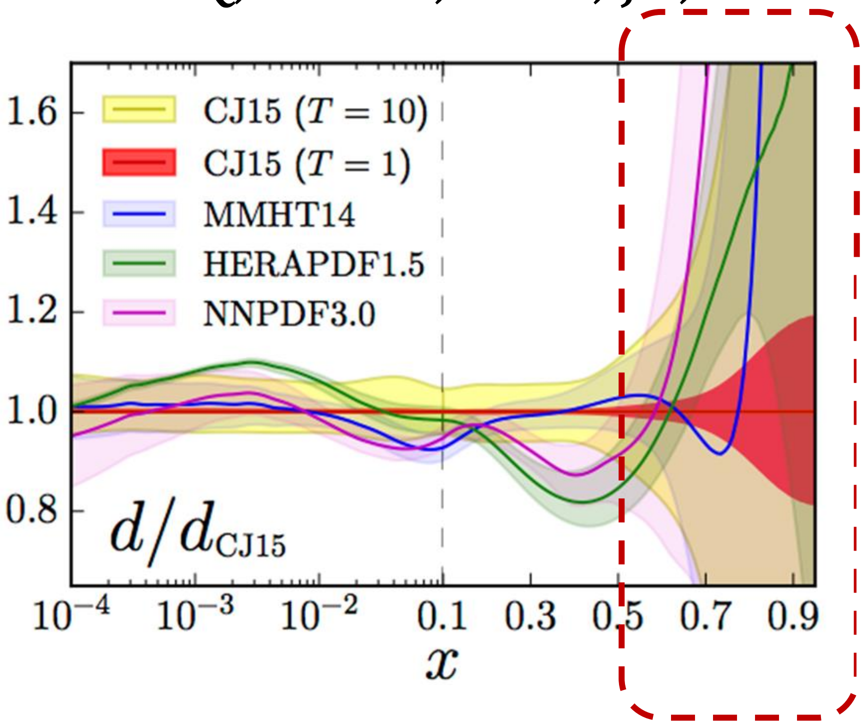
- Assumptions imposed

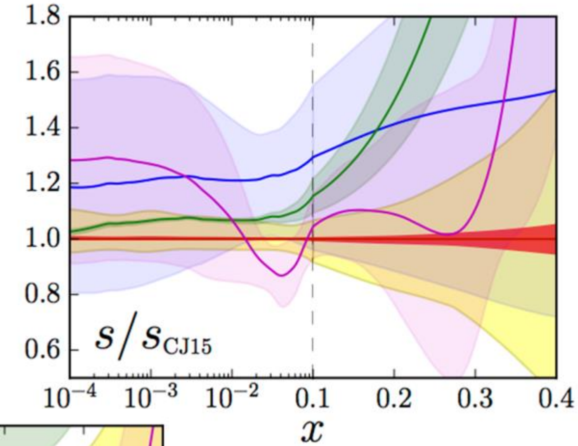
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

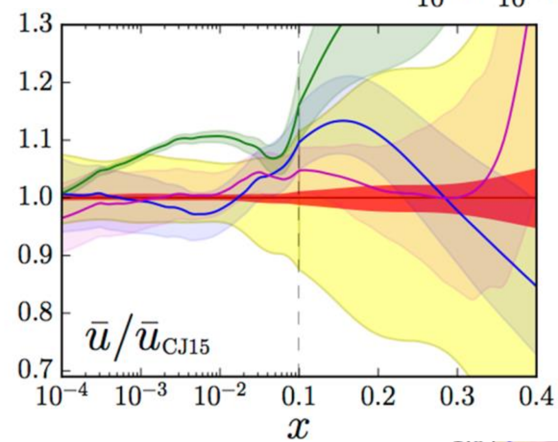
$$s = \bar{s} = \kappa\left(\bar{u} + \bar{d}\right)$$

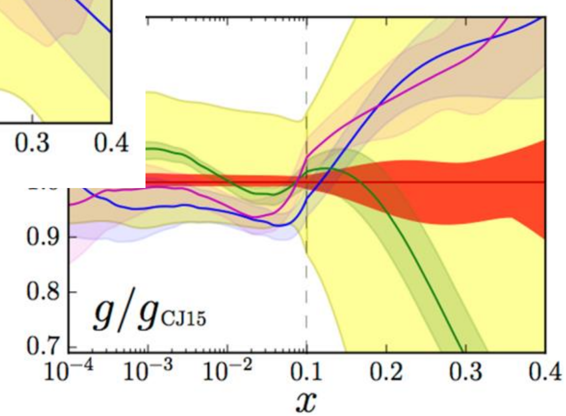
Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.







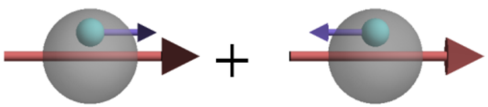


CTEQ-JLAB
https://www.jlab.org/theory/cj/

PDFs on the Lattice

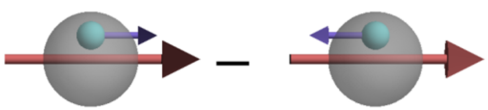
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

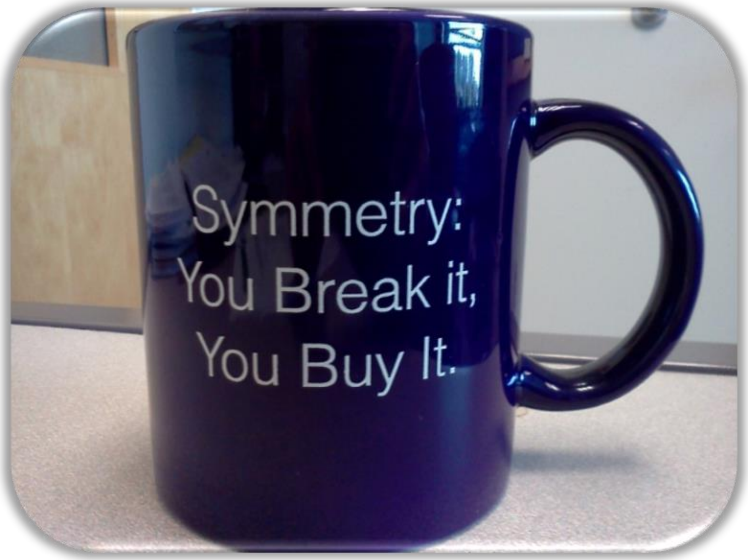
§ True distribution can only be recovered with all moments

PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Novel proposals to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea

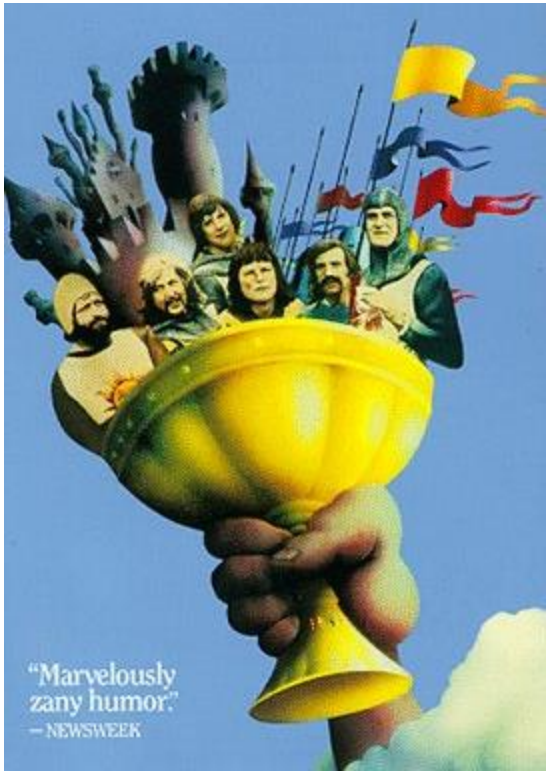
W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

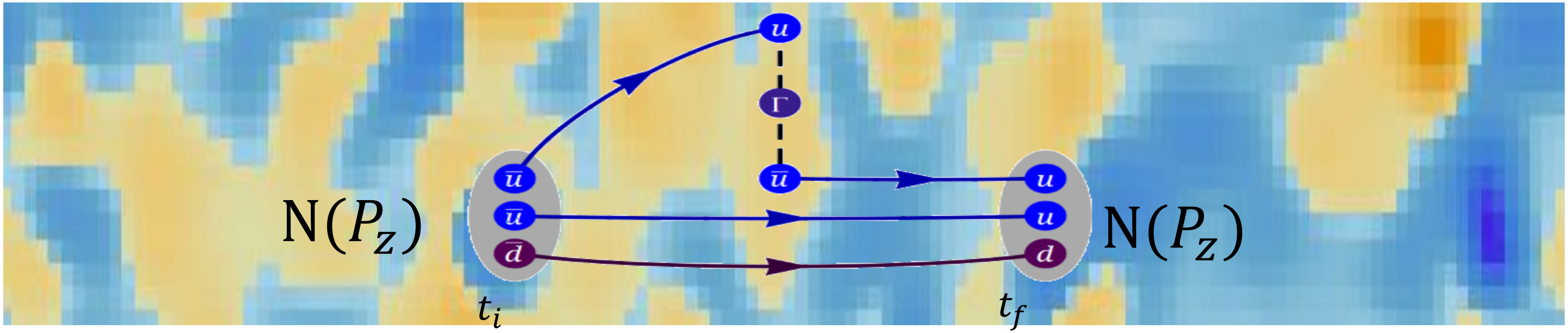
They carry secret plans to build the most powerful

Bjorken-x Dependent Hadron Structure

Biased selected results, highlighting work done by MSU students/postdocs

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P\right\rangle$$

§ Recover true distribution (take Pz → ∞ limit)

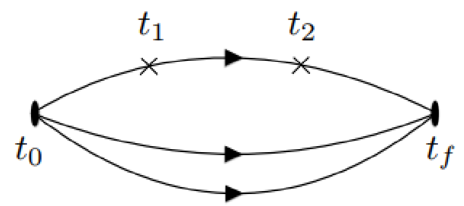
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)P_z)^2}\right)$$

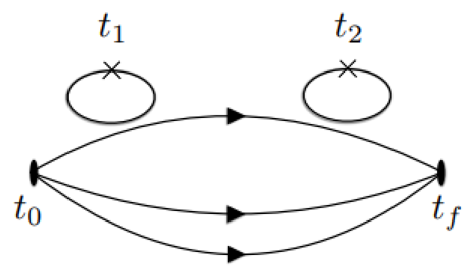
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

§ Short-distance factorization (SDF)

- **pseudo-PDF** method (A. Radyushkin, 2017)
- Hadronic tensor currents
  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)
  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions
  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)





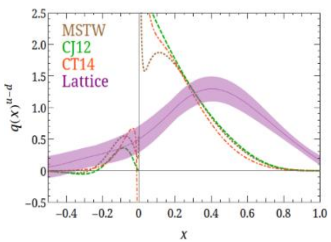
Quantities that can be calculated on the lattice today $=$ $\sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

Lattice Parton Calculations

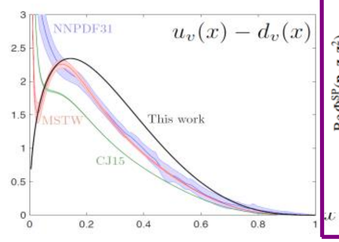
§ Rapid developments!

HL, Few Body Syst. 64 (2023) 3, 58

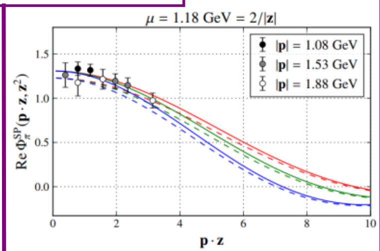
First unpol. PDF lattice calculation



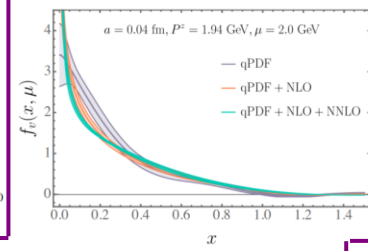
First lattice pseudo-PDFs



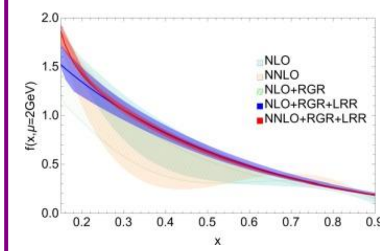
Euclidean correlation functions



1st NNLO PDF

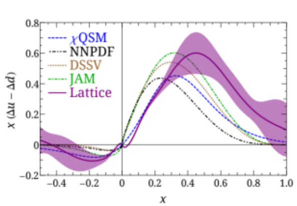


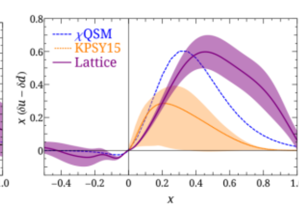
1stPDF w/ LRR+RGR



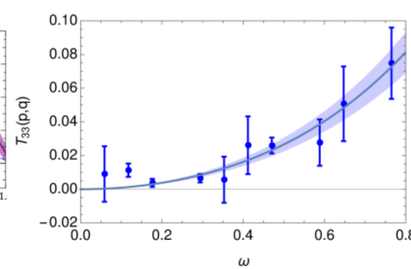
2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024

Pol. PDFs and mass corrections

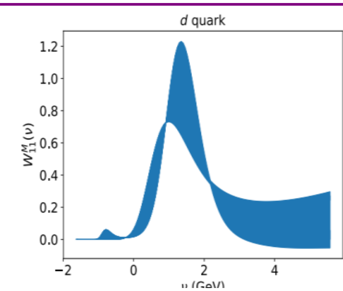




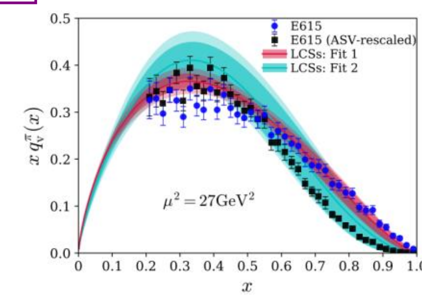
Compton amplitude



Hadronic tensor

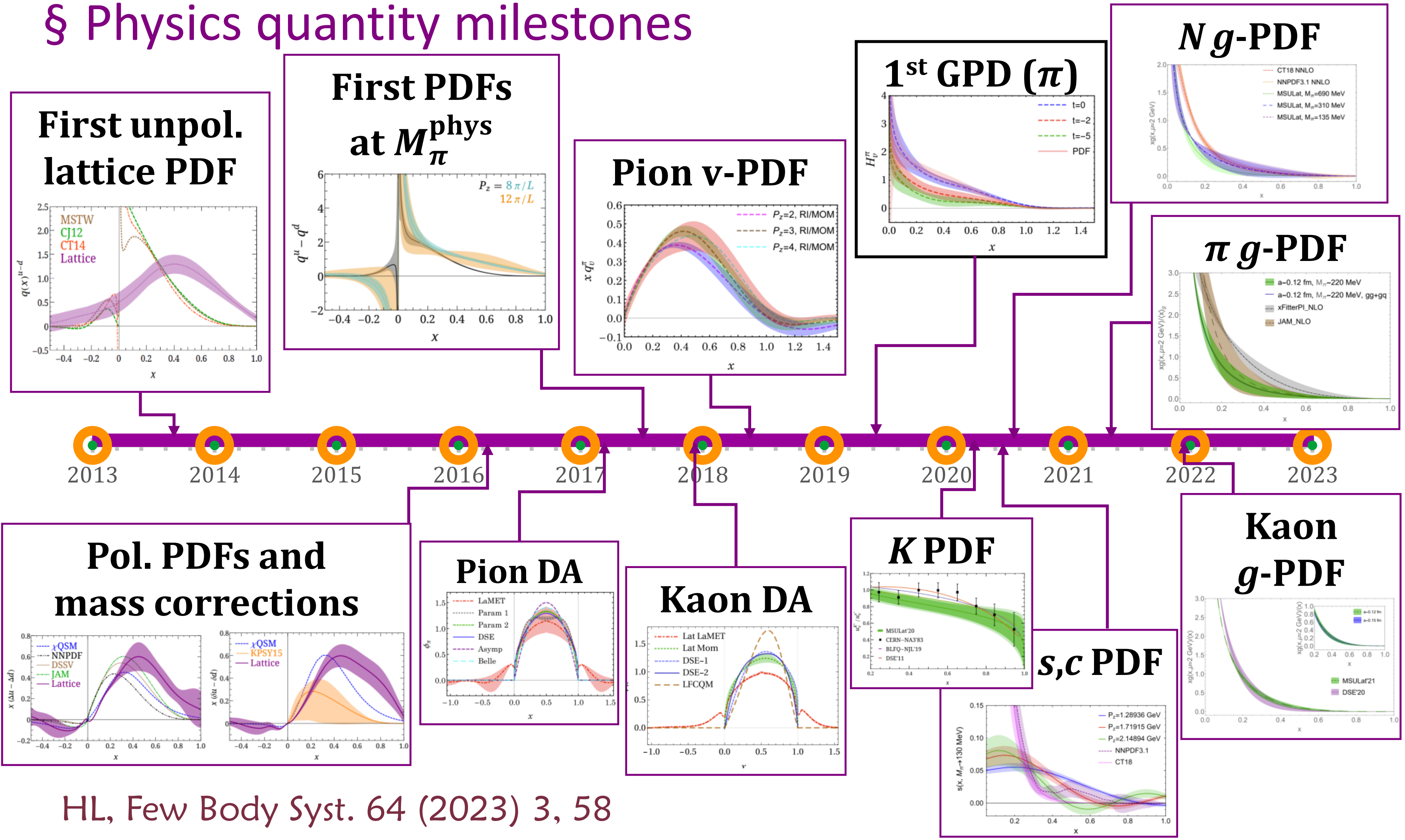


LCS



Lattice Parton Calculations

§ Physics quantity milestones

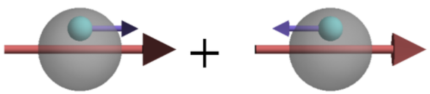


HL, Few Body Syst. 64 (2023) 3, 58

Lattice Example Results

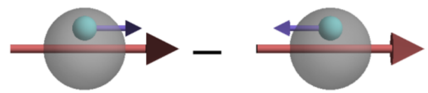
§ Summary of physical pion mass PDFs results

unpolarized



$u(x) - d(x)$

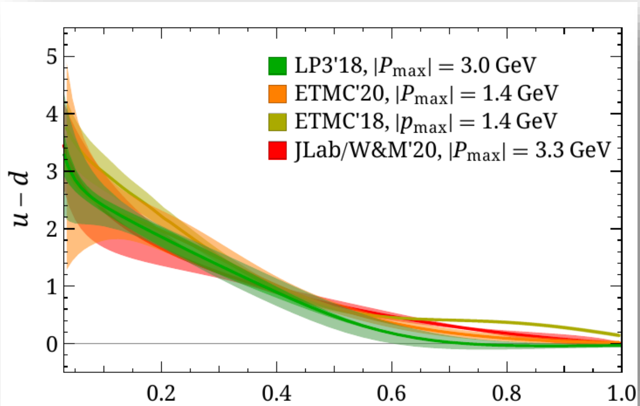
longitudinally polarized

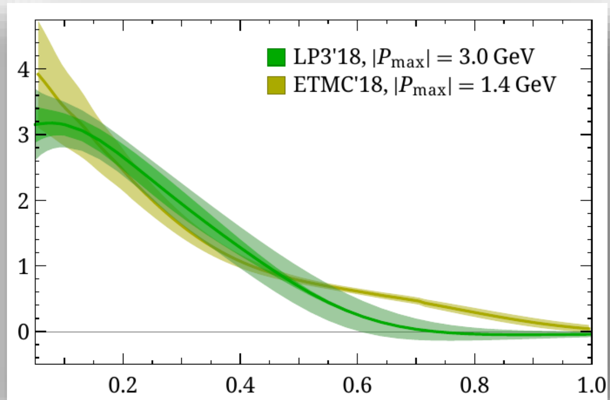


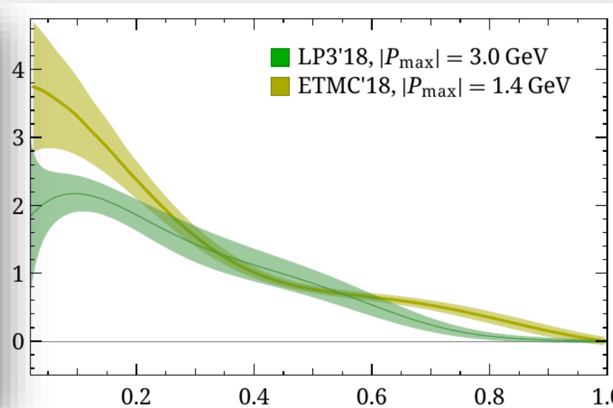
$\Delta u(x) - \Delta d(x)$

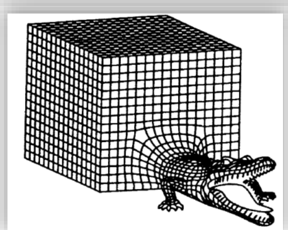
transversely polarized

$\delta u(x) - \delta d(x)$







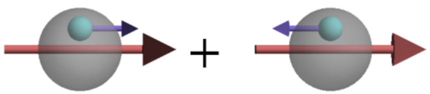


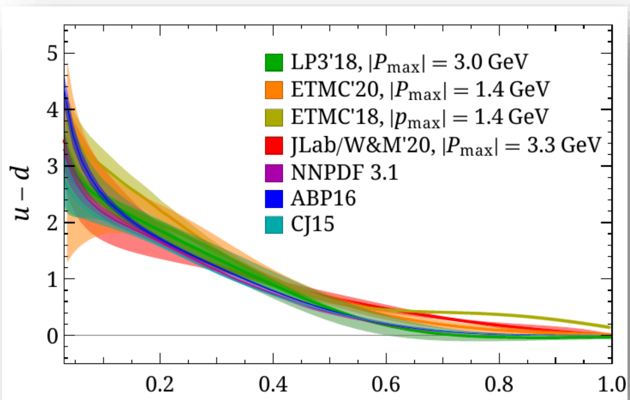
Finite volume,
Discretization,
...

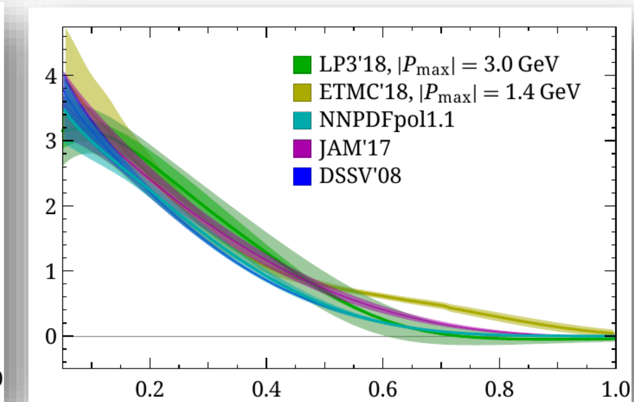
2006.08636 (PDFLattice2019)

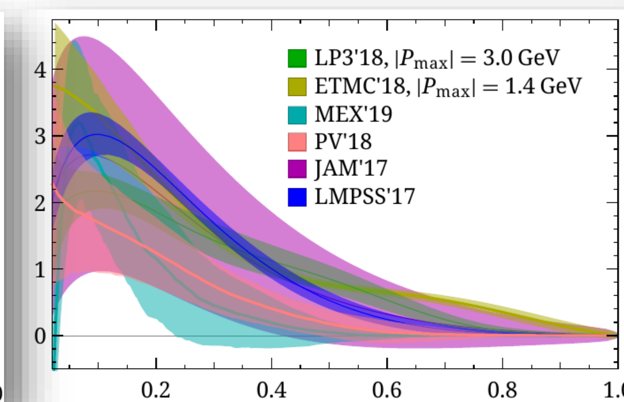
Lattice Example Results

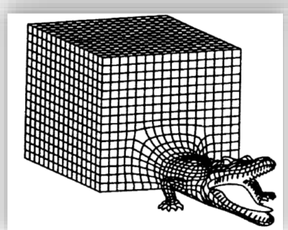
§ Summary of physical pion mass PDFs results

unpolarized	longitudinally polarized	transversely polarized
		
$u(x) - d(x)$	$\Delta u(x) - \Delta d(x)$	$\delta u(x) - \delta d(x)$









Finite volume,
Discretization,
...

2006.08636 (PDFLattice2019)

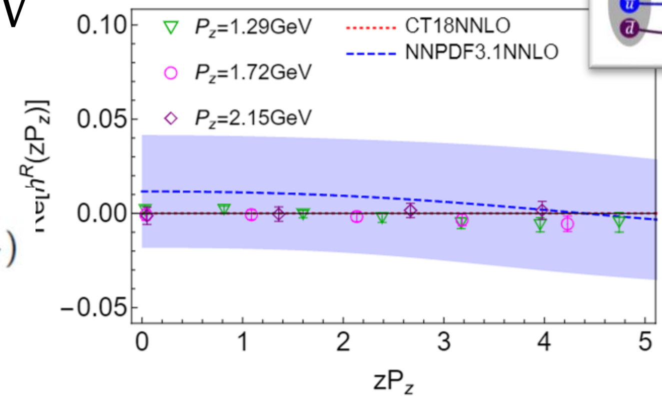
First Lattice Strange PDF

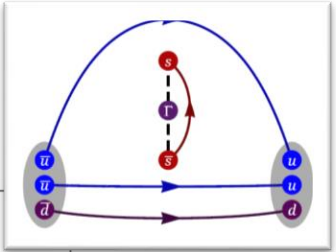
§ Results by MSULat/quasi-PDF method

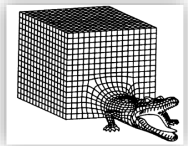
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\, (s(x) - \bar{s}(x)) \cos(xzP_z)$$





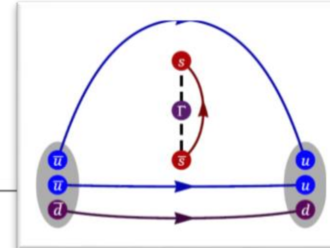


Quantities that can be calculated on the lattice

First Lattice Strange PDF

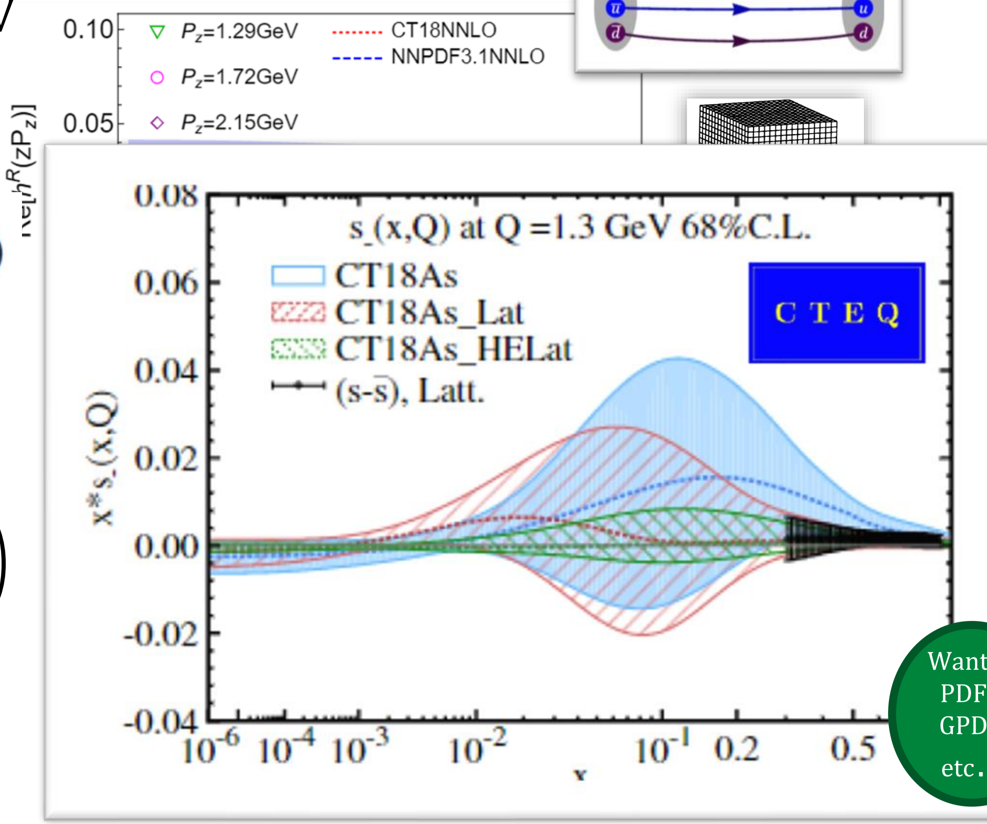
§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV



2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$



§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2204.07944

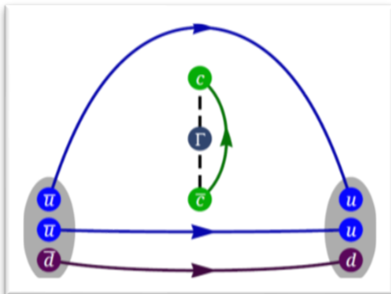
Wanted PDFs, GPDs, etc…

§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

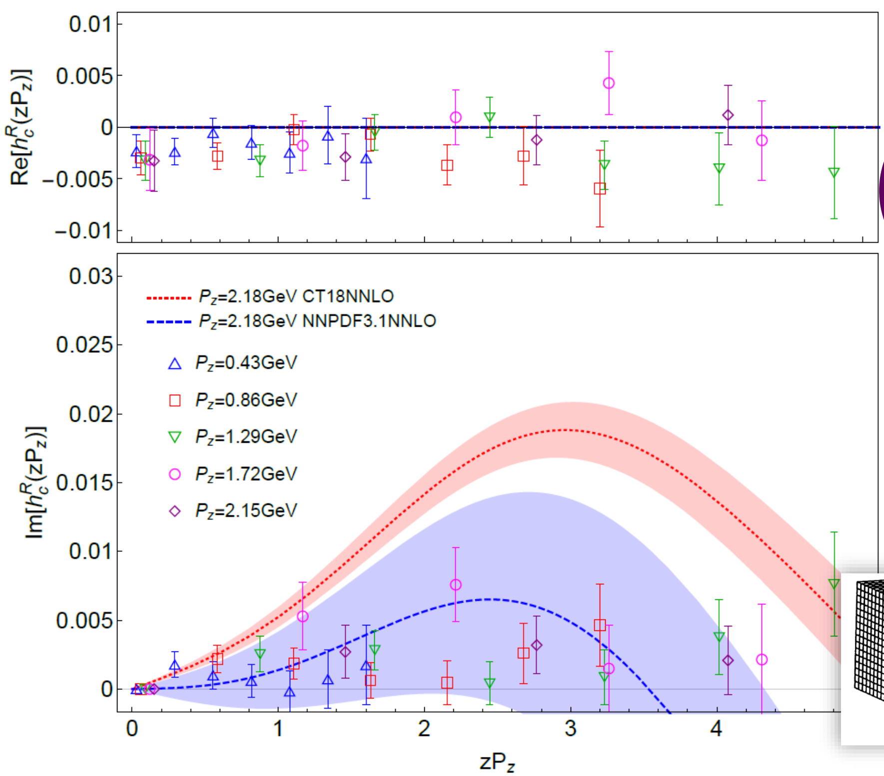
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

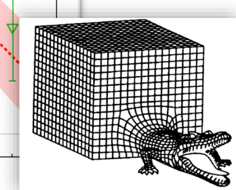
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF

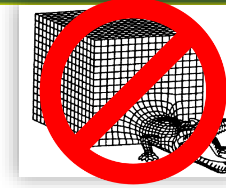


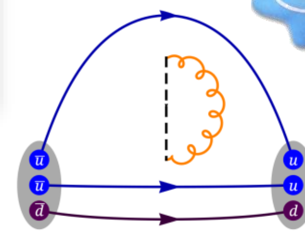
Quantities that can be calculated on the lattice



Gluon PDF in Nucleon

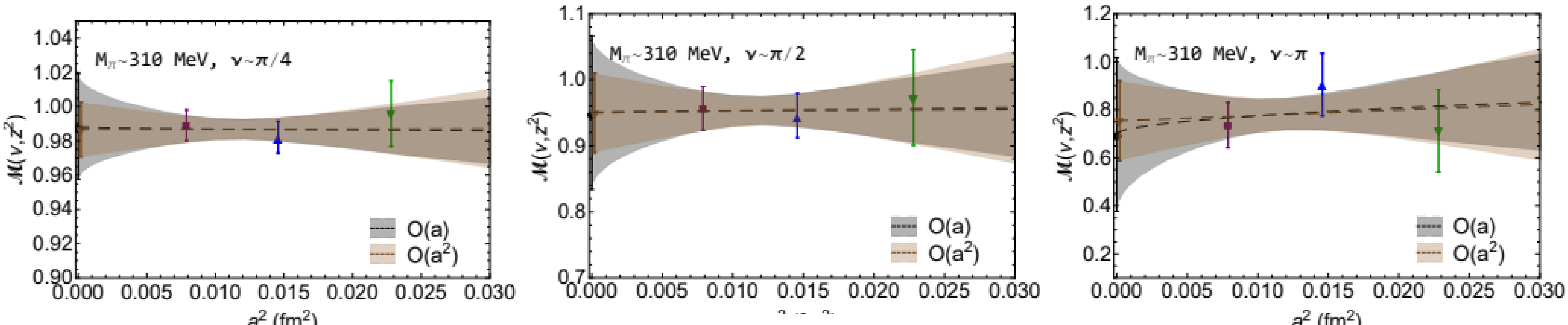
§ Continuum Gluon PDF w/ pseudo-PDF

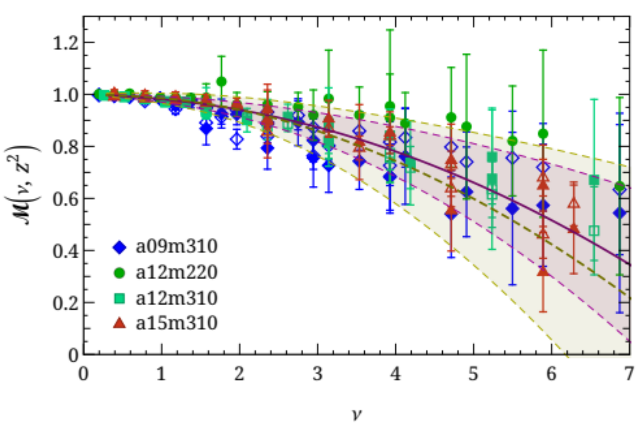


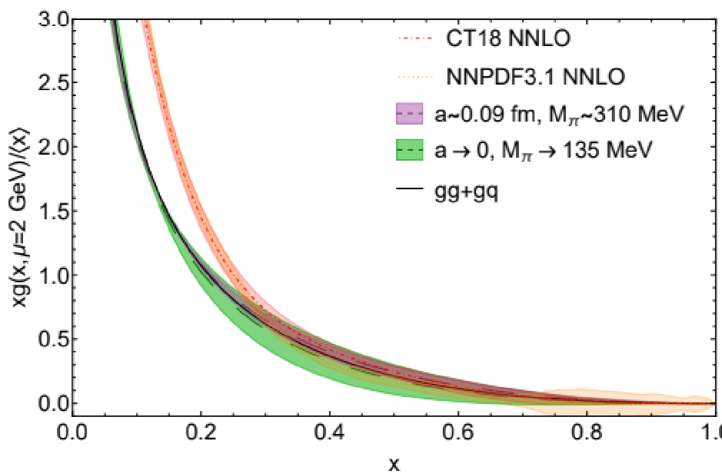


- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

arXiv:2210.09985







G: Bill Good

Nucleon Tomography

- Assuming we live in the Marvel Universe
  - The special quantum tunnel allows us to shrink to the size particle to sub-nucleon scale ($< 10^{-15}$m)
- What would it look like to travel inside the nucleon?

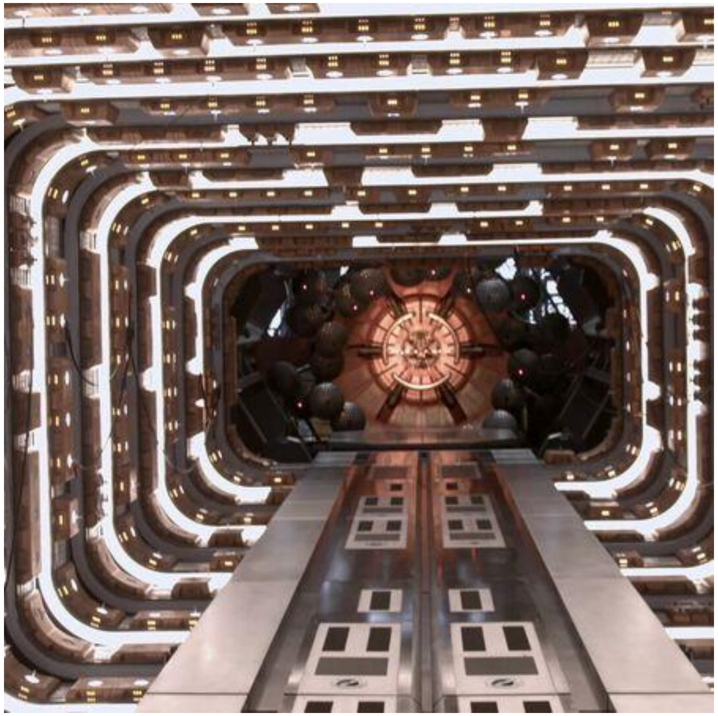
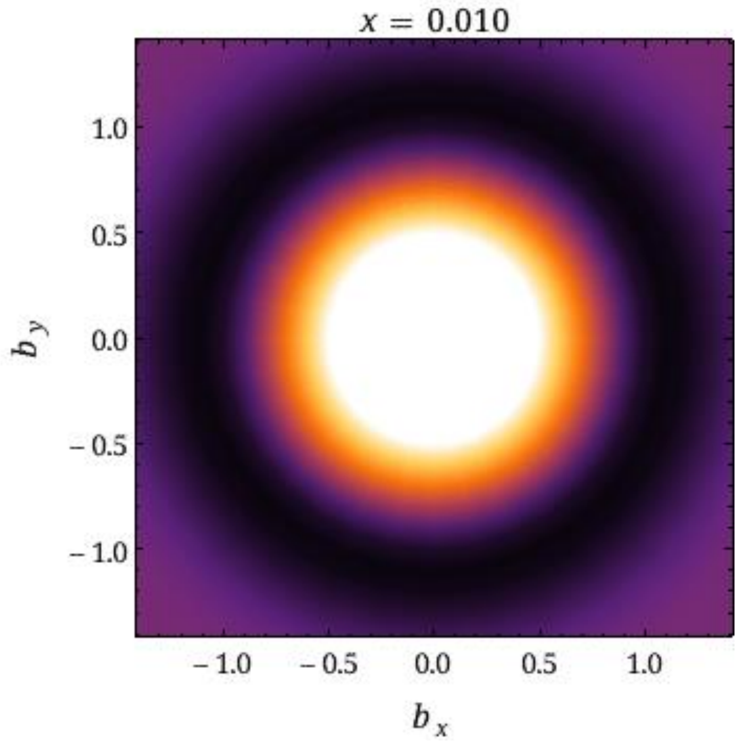


Image credit: Marvel Studios

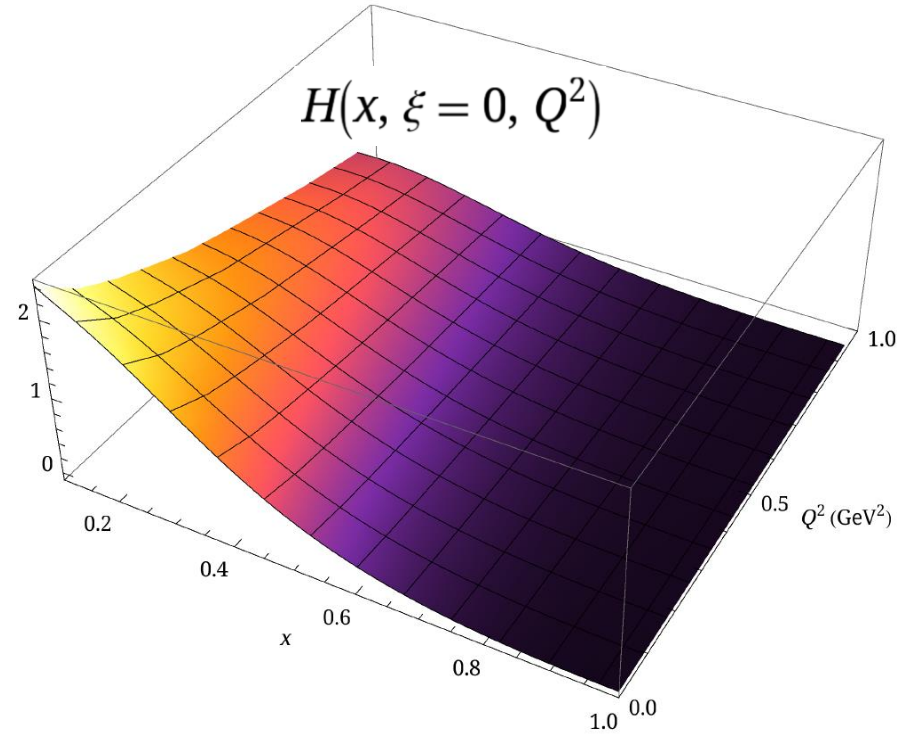


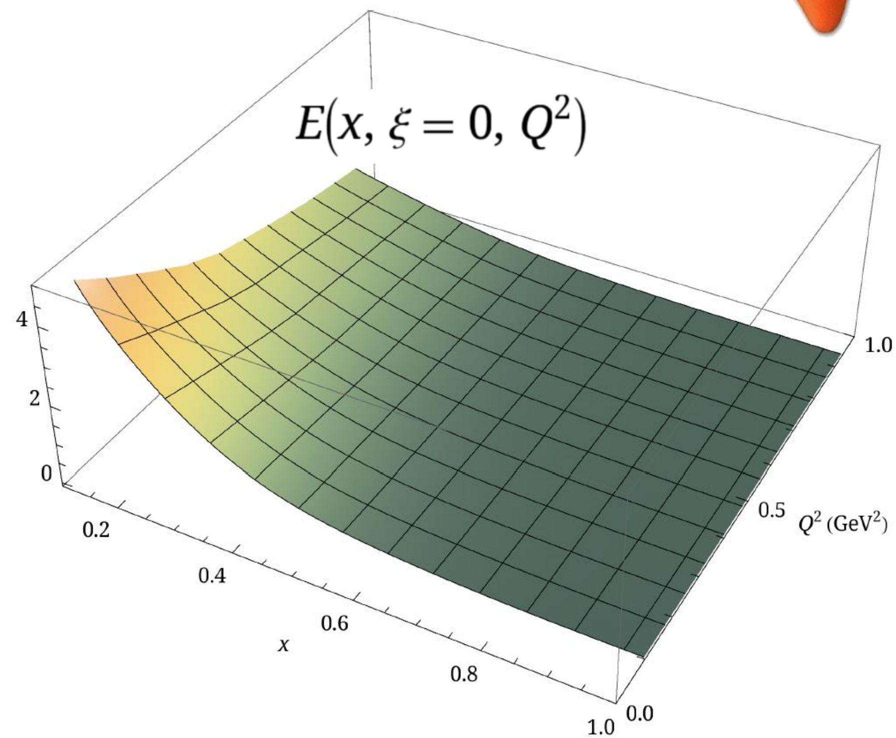
Thanks to Cottrell Scholar Award from RCSA

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results





HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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- Lattice details: clover/2+1+1 HISQ (MSULat)
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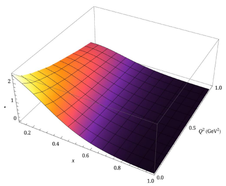
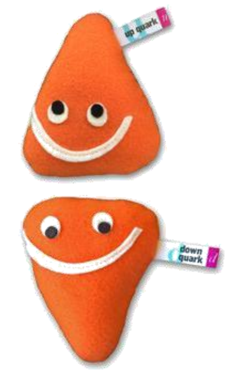
- $\xi = 0$ isovector nucleon GPD results

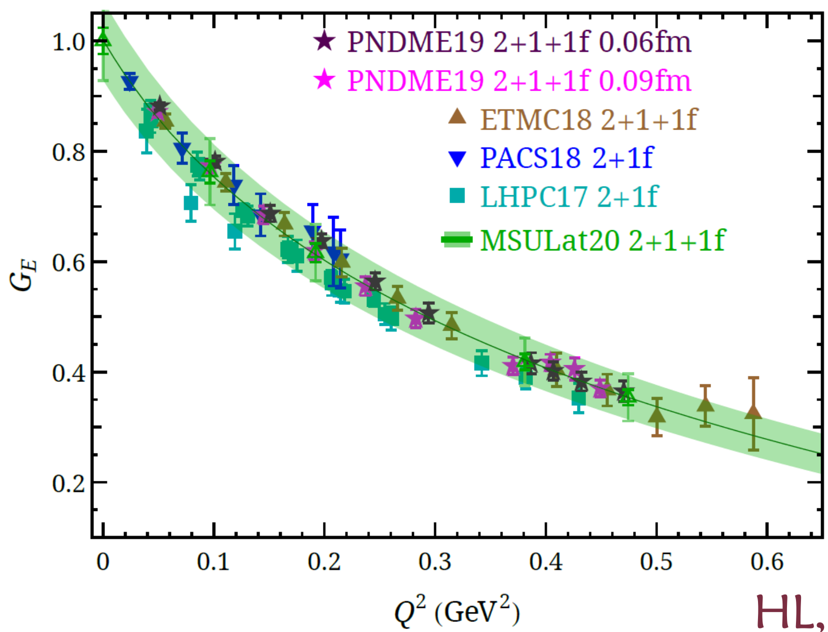
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\text{ even}}$$



$n = 1$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon GPDs

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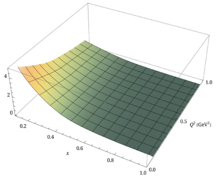
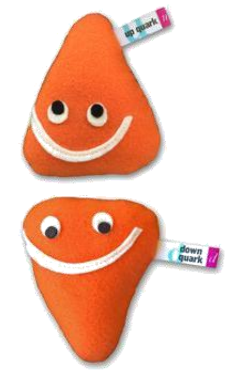
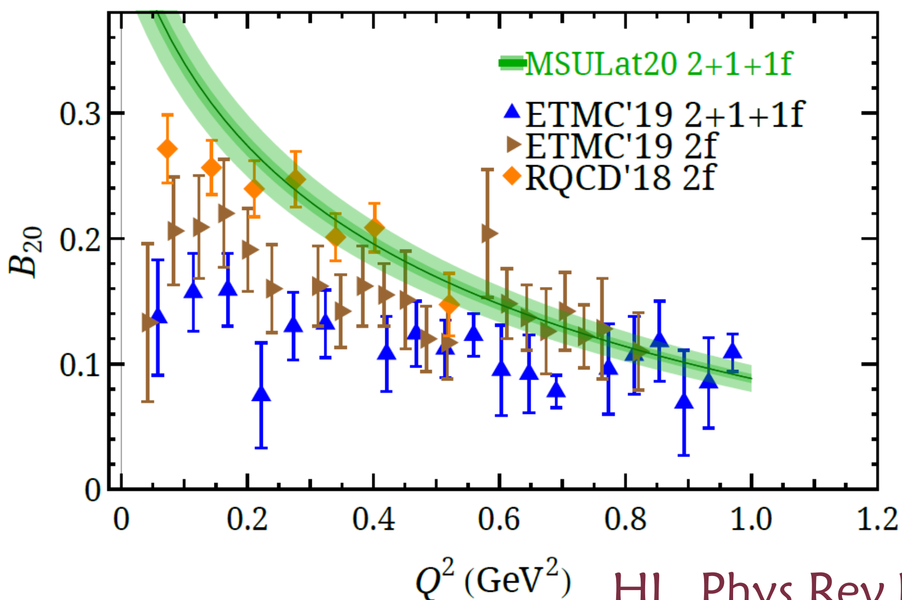
$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$



$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

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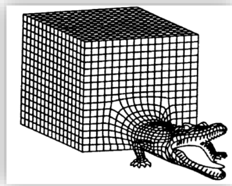
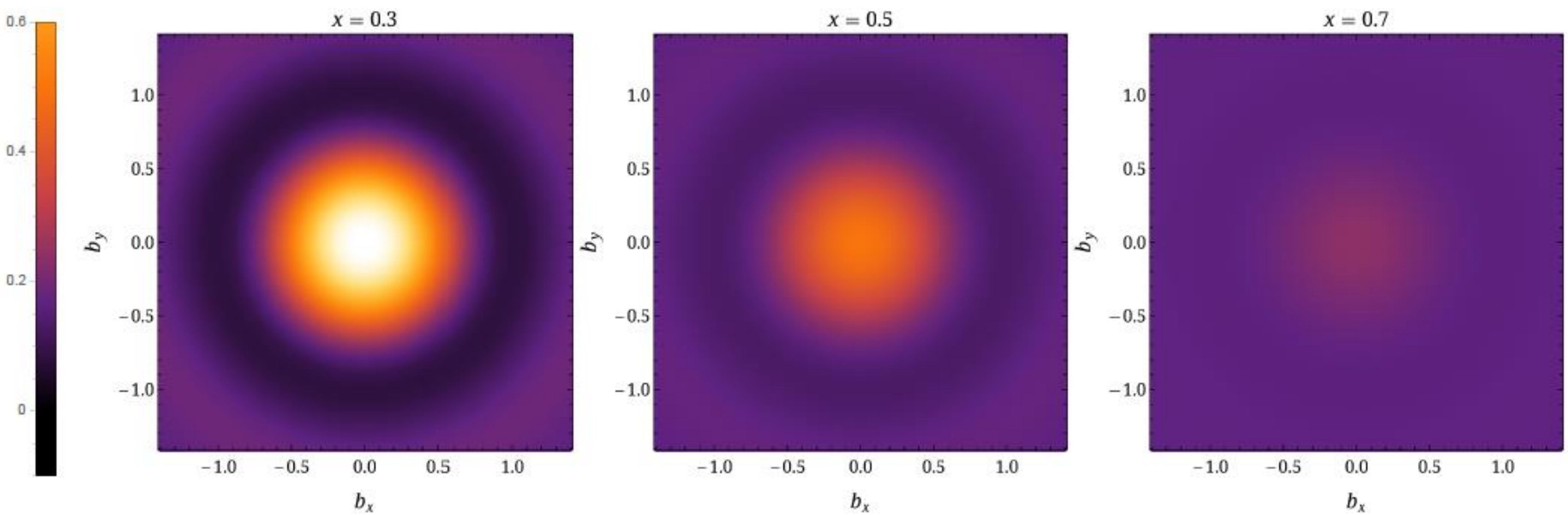
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^{\,2}) e^{i\vec{q}\cdot\vec{b}}$$



finite-volume,
discretization,



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Challenges

§ Large momentum is essential

- With sufficient statistics nucleons may reach 5 GeV

§ Renormalization of linear divergence

- Wilson-line ops have linear divergences that must be subtracted

§ Methods for signal-to-noise improvement

- Gluonic observables, new ideas for large momentum

§ Inverse problems PDF extraction in SDF

- Remove the model/preconditioner-choice dependence

§ Reaching long-range correlations in LaMET

- For small-x physics, new methods for calculating longer-range correlations must be developed

Whitepaper: Lattice QCD Calculations of Parton Physics, 2202.07193

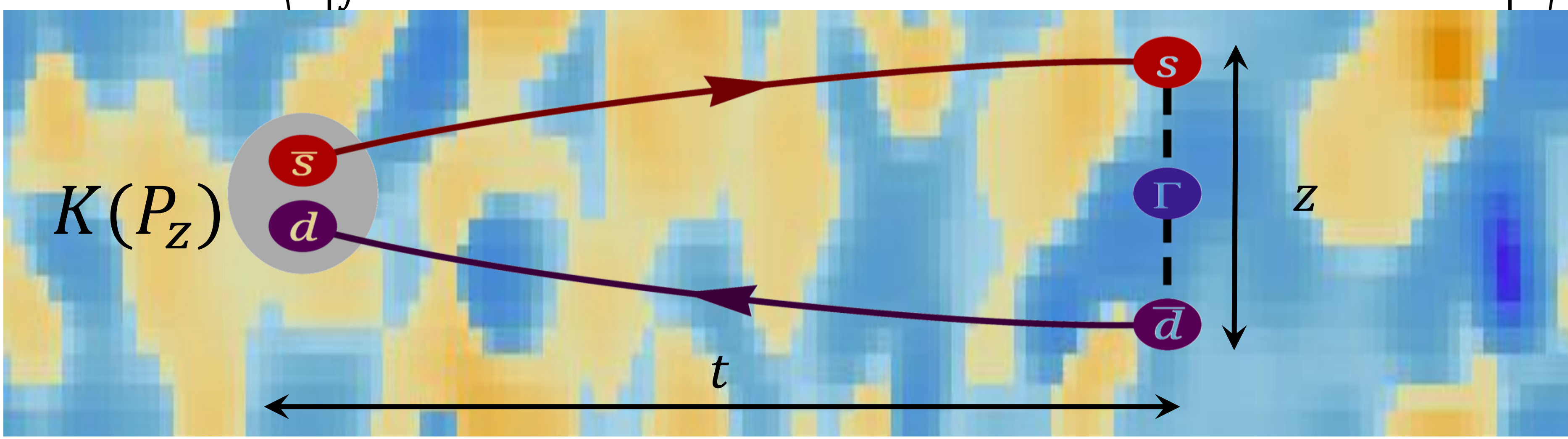
Application on Inverse Problem

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculat ed kernel

R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Example: Pion/Kaon Distribution Amplitude

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$

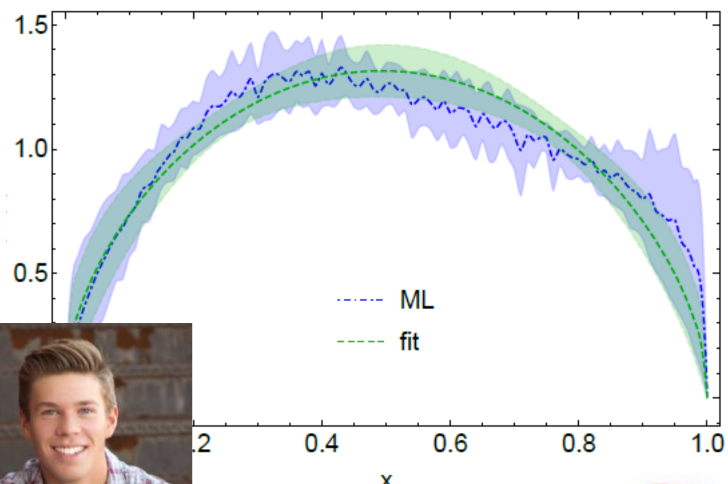


Application on Inverse Problem

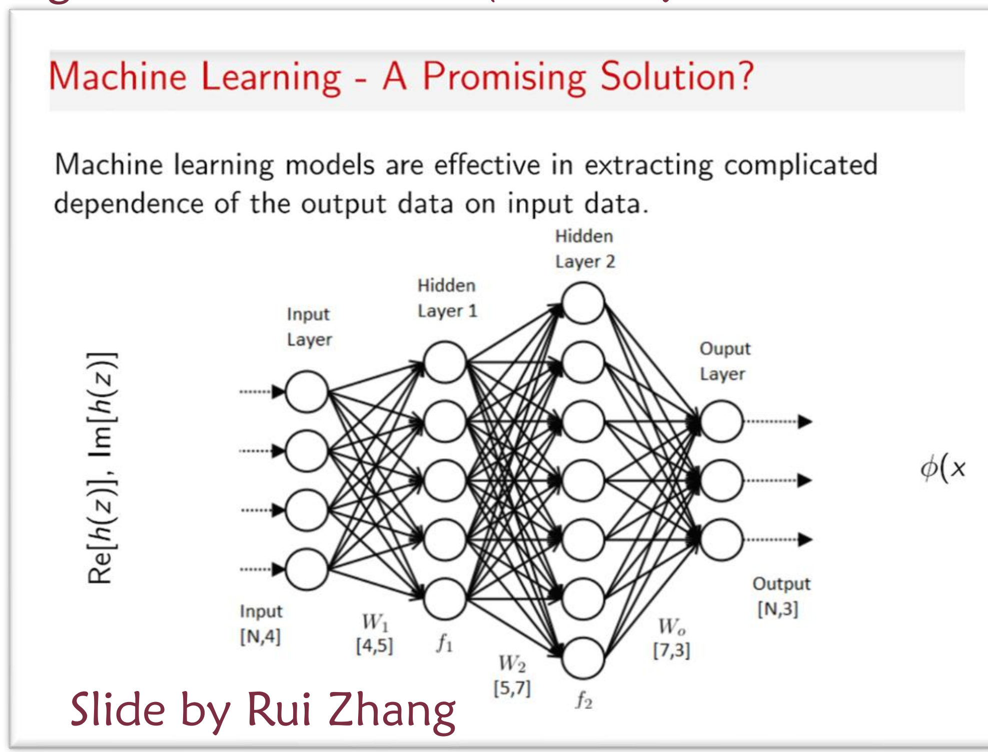
Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Pion Distribution Amplitude



UG: Carson Honkala



Slide by Rui Zhang

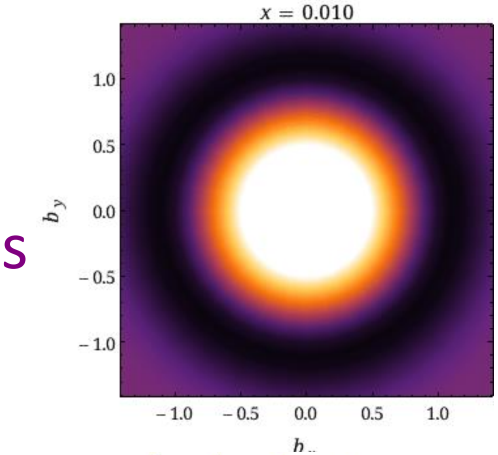
Summary and Outlook

§ Exciting era using LQCD to study pion and nucleon structure

- Well-studied systematics → precision structures

§ Overcoming longstanding obstacles

- Bjorken-x dependence of parton distributions are widely studied with LaMET and its variants

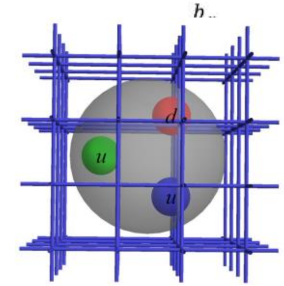


§ Precision and progress are limited on resources

- Challenges = new opportunities quantities

§ In the future

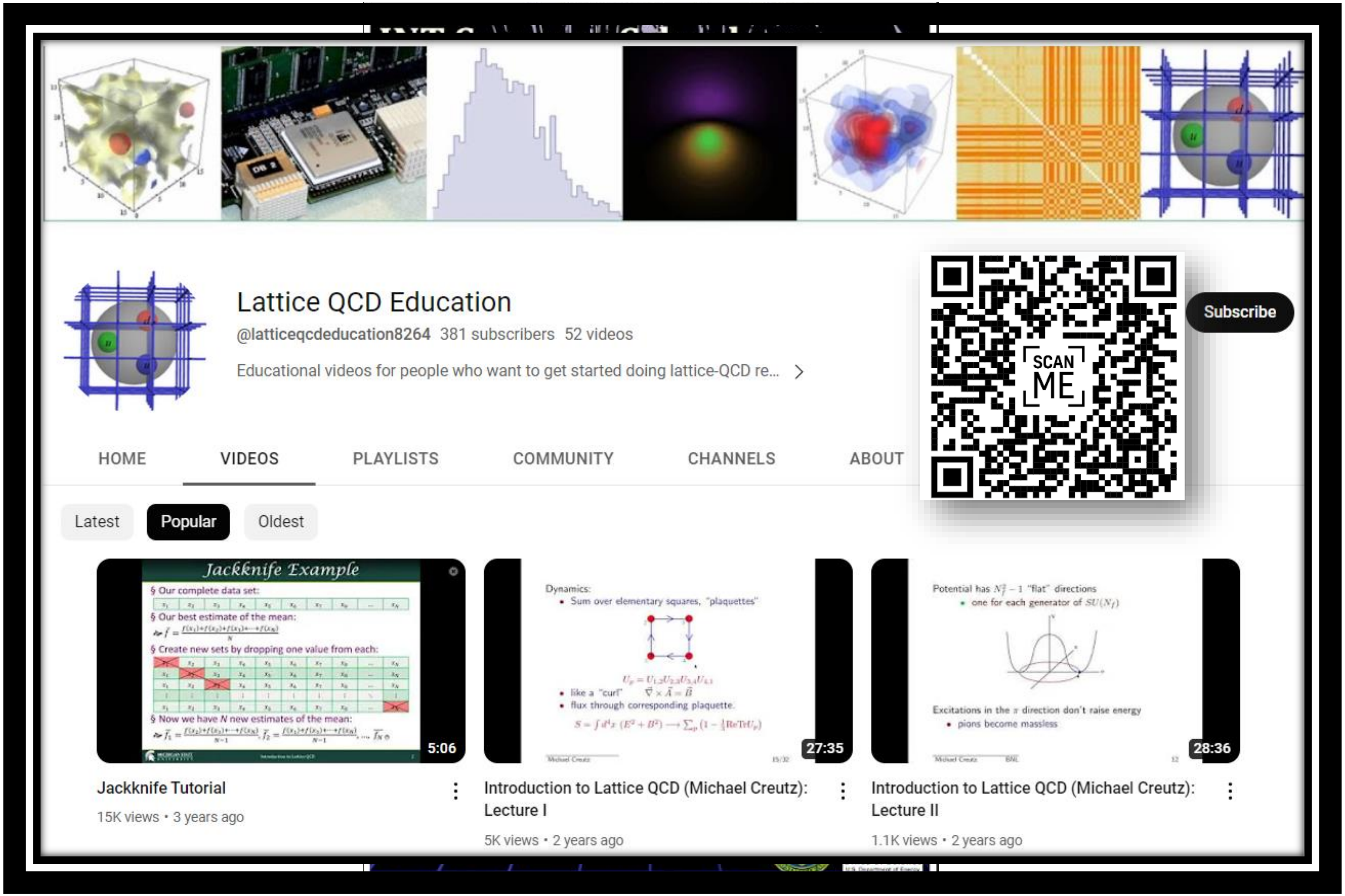
Theory Input → **Global Analysis of GPDs** ← **Exp't Input** +



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
The work of HL is sponsored by NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 & RCSA Cottrell Scholar Award

Interested in learning more?



If time allows…

Learning by Doing!

- Prior Python with Jupyter notebook experience required
  - How many of you have not used these before?
- Form groups of about 4 students
- Introduce yourself to the other students in your group (5-10 mins)
  - Name and preferred pronoun
  - If you could hang out with any cartoon character, who would you choose and why?
  - **Make sure you listen to each other!**

Hands-on Exercise

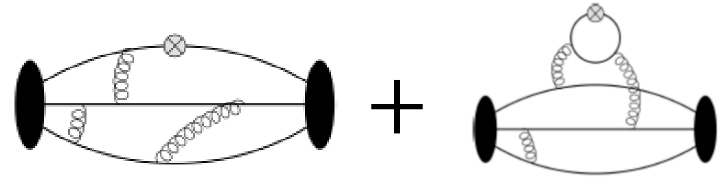
§ Hands-on exercise

- Always starts with “pre-class” notebook before moving on to “in-class” one
  - Set01: Jackknife analysis (Mon)
  - Set02: Calculating proton masses (Mon)
  - Set03: Extracting nucleon charges (Tue)

Backup Slides

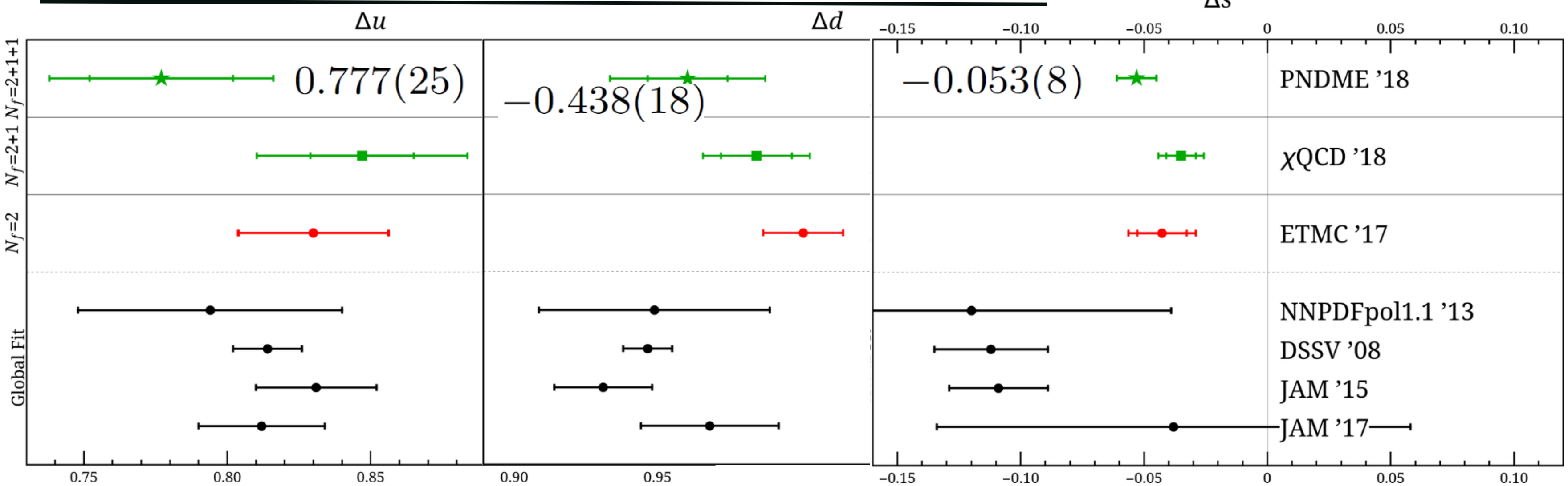
Quark Spin Contribution

§ Total quark contributions



PNDME	$g_A^u \equiv \Delta u$	$g_A^d \equiv \Delta d$	$g_A^s \equiv \Delta s$
Connected	0.895(21)	−0.320(12)	
Disconnected	−0.118(14)	−0.118(14)	−0.053(8)
Sum	0.777(25)	−0.438(18)	−0.053(8)
ETMC	0.830(26)	−0.386(18)	−0.042(10)(2)
χQCD	0.847(18)(32)	-0.407(16)(18)	-0.035(6)(7)

Difference caused by Δq^{disc}



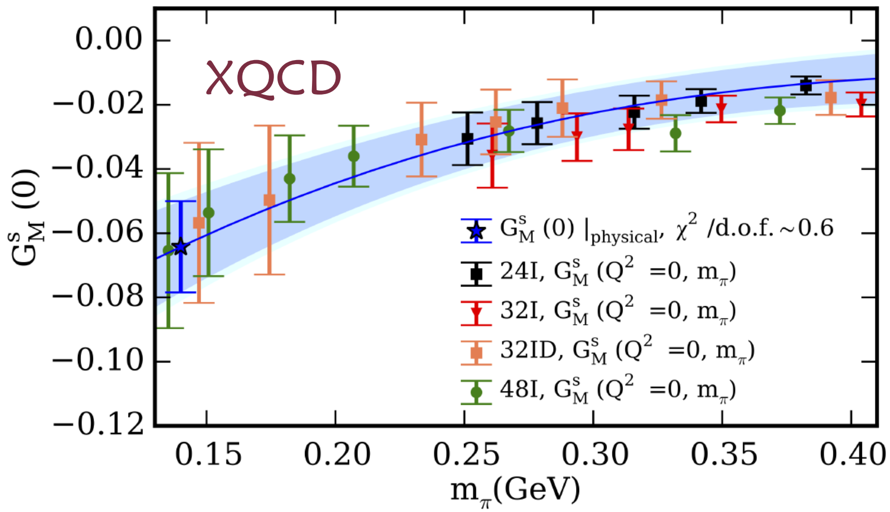
$$\sum_{q=u,d,s}\left(\tfrac{1}{2}\Delta q\right) = 0.143(31)$$

PNDME, 1806.09006, 1806.10604

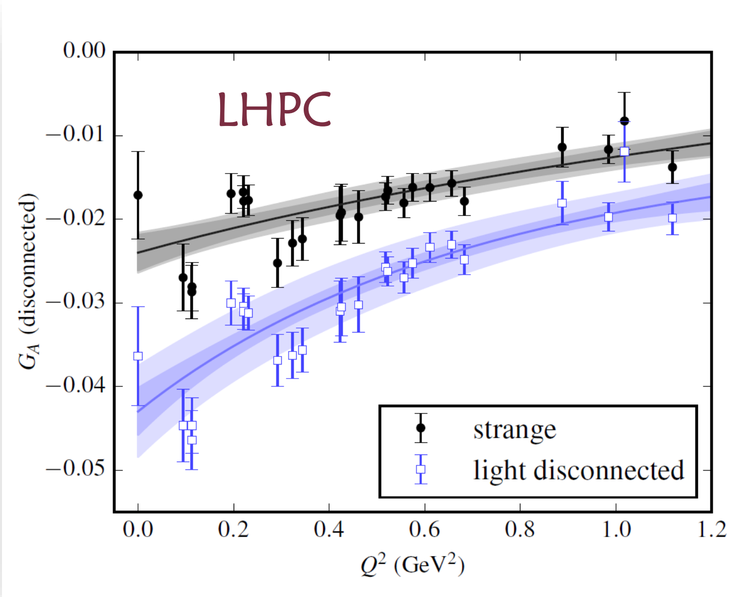
The work of HL is supported by NSF CAREER Award under grant PHY 1653405

Other Form Factors

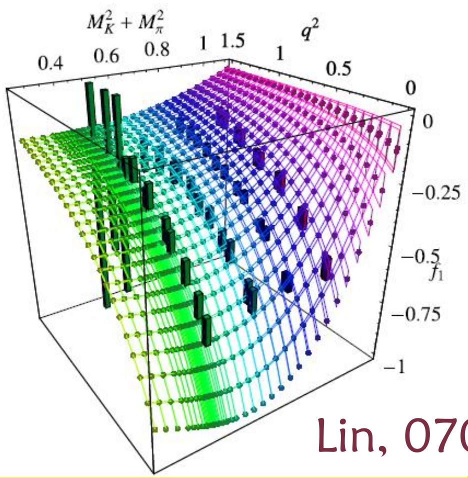
§ Toward flavor-dependent nucleon form factor



Sufian et al., Phys. Rev. Lett. 118, 042001 (2017)



Green et al., Phys. Rev. D 95, 114502 (2017)



Lin, 0707.3844 [hep-lat]

§ Hyperon form factor

$\bar{\nu}_\mu p \to \mu^+ \Lambda^0$

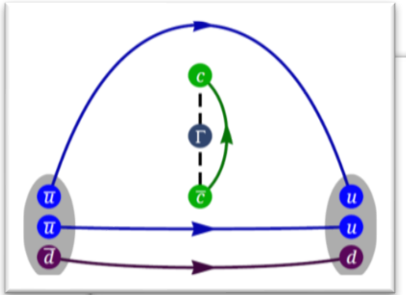
$\bar{\nu}_\mu n \to \mu^+ \Sigma^-$

$\bar{\nu}_\mu p \to \mu^+ \Sigma^0$

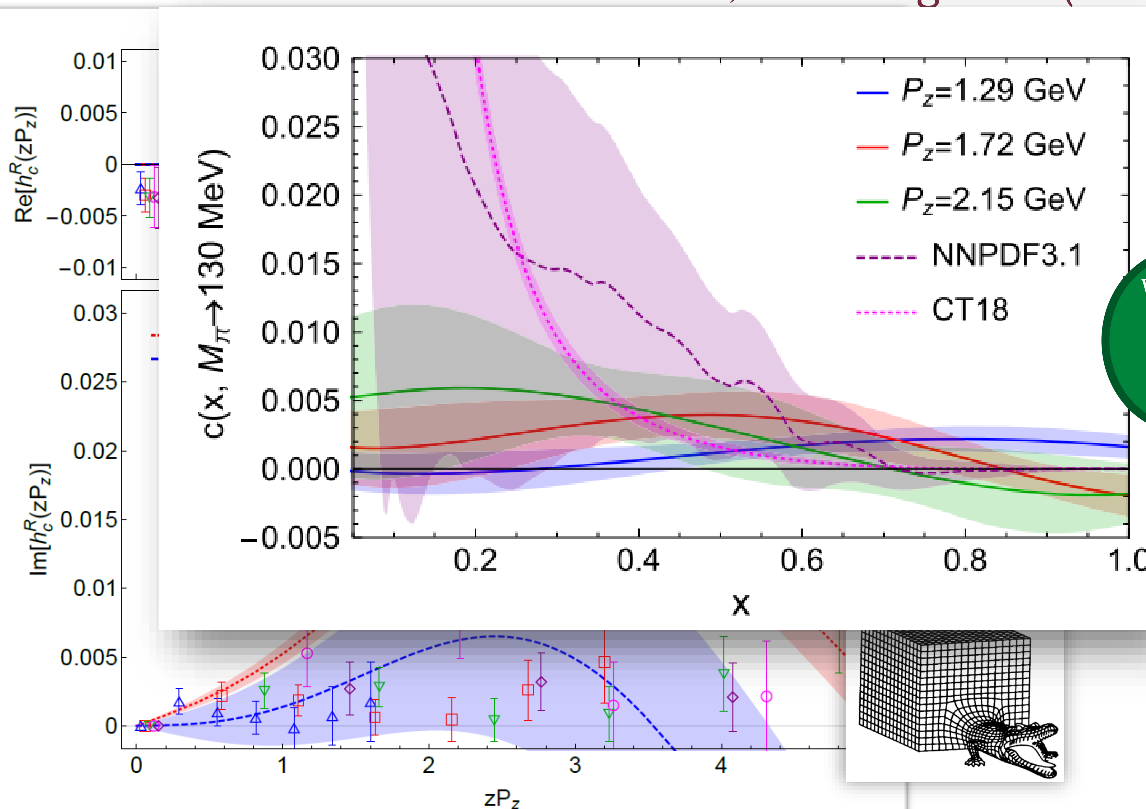
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c-\bar{c}$ distribution
- much smaller than strange PDF



Wanted PDFs, GPDs, etc…

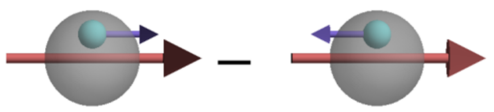
Moments of PDFs

§ PDG-like rating system or average

§ LatticePDF Workshop

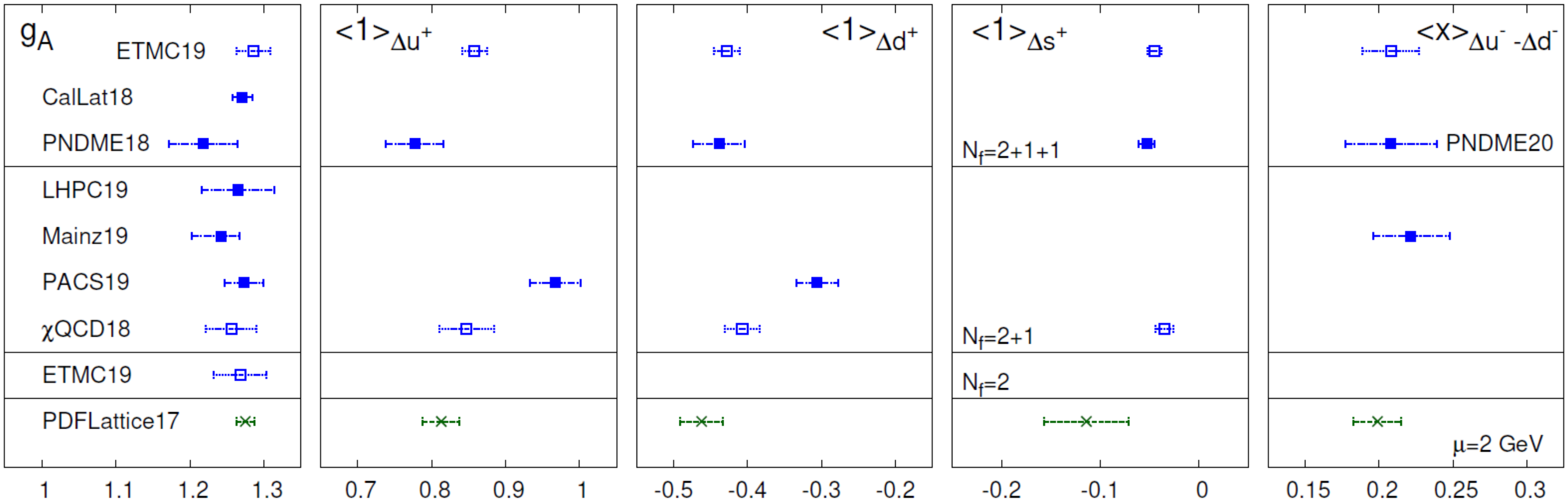
- Lattice representatives came together and devised a rating system

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1}\,\Delta q(x)$$



§ Recent lattice QCD/global fit status

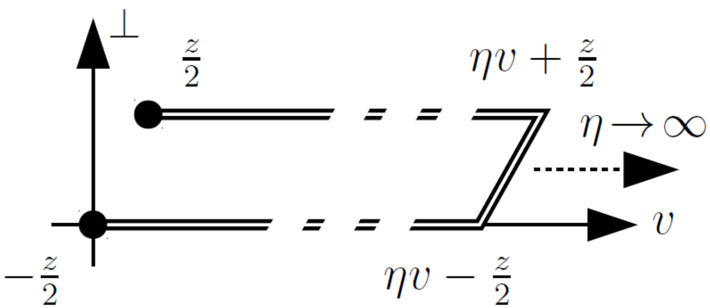
LatticePDF Report, 1711.07916,2006.08636



Orbital Angular Momentum

§ Two definitions: Ji vs Jaffe & Manohar

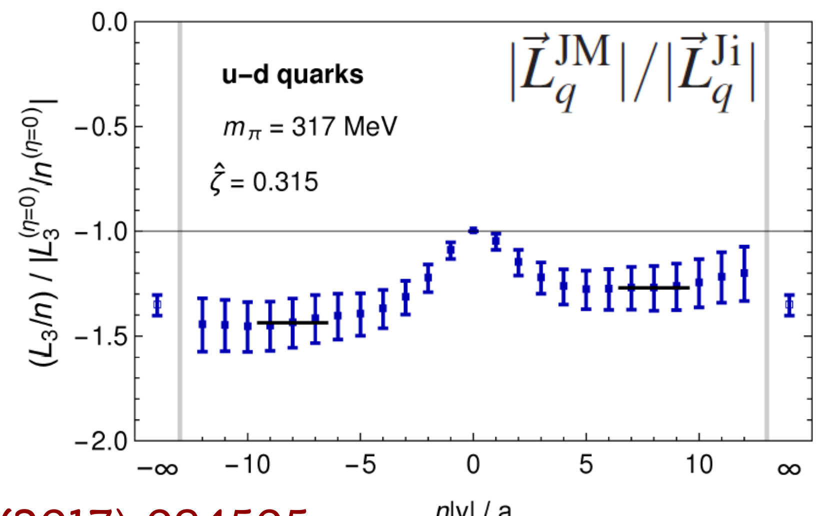
$$\vec{L}_q^{\mathrm{Ji}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{D}\right] q \qquad \vec{L}_q^{\mathrm{JM}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{\nabla}\right] q\,;$$



- $\eta = 0$ gives Ji's OAM
- Staple $\eta \to \infty$ gives Jaffe-Manohar OAM
- Difference is accumulated torque from final-state interaction

§ First result carried out by M. Engelhardt

2+1f clover at 518-MeV pion mass



Phys. Rev. D95 (2017) 094505

New Interactions in Beta Decays

§ Neutron beta decay could be sensitive to new interactions:

$$H_{\text{eff}} = G_F \left(J_{V-A}^{\text{lept}} \times J_{V-A}^{\text{quark}} + \sum_i \varepsilon_i^{\text{BSM}} \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right)$$

$$\hat{O}_S = \overline{u}d \times \overline{e}(1-\gamma_5)\nu_e \quad \rightarrow \quad g_S = \langle n|\overline{u}d|p\rangle$$

$$\hat{O}_T = \overline{u}\sigma_{\mu\nu}d \times \overline{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e \quad \rightarrow \quad g_T = \langle n|\overline{u}\sigma_{\mu\nu}d|p\rangle$$

- ε_S and ε_T are related to the masses of the new TeV-scale particles (just like $G_F \propto M_{W,Z}^{-2}$)

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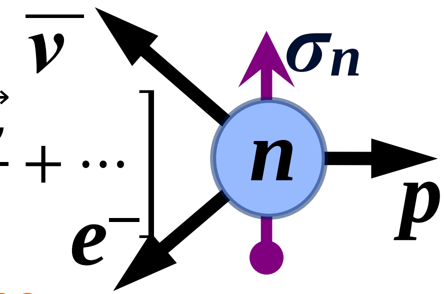
- ε_S and ε_T are related to the masses of the new TeV-scale particles
- Parameters sensitive to new physics



$$d\Gamma \propto F(E_e)\left[1 + A\frac{\vec{\sigma_n}\cdot\vec{p_e}}{E_e} + b\frac{m_e}{E_e} + \left(B_0 + B_1\frac{m_e}{E_e}\right)\frac{\vec{\sigma_n}\cdot\vec{p_\nu}}{E_\nu} + \cdots\right]$$

Fierz interference term:
Deviations from the leading-order e^- spectrum

Energy-dependent part of the **neutrino asymmetry parameter** with neutron spin

$$\{b,B\}_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$$

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Precision LQCD input ($m_\pi \approx 140$ MeV, $a \to 0$)

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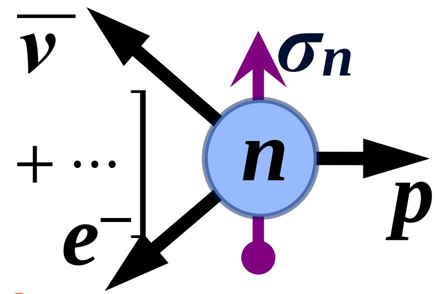
- ε_S and ε_T are related to the masses of the new TeV-scale particles
- Paramete

$d\Gamma \propto F(E_e)$ ... $+ \cdots$



Ongoing and Future Experiments	Expected Precision
UCNb & UCNB at LANL	10^{-3} to 10^{-4}
Nab at ORNL	10^{-3}
FRMII in Munich, …	
CENPA ^{6}He(b_{GT})	10^{-3} to 10^{-4}

Fierz inter

Deviations f

leading-ord

he

arameter

$\{b,B\}_{\text{BSM}} = f(\varepsilon_{S,T}\, g_{S,T})$

Precision LQCD input

($m_\pi \approx 140$ MeV, $a \to 0$)

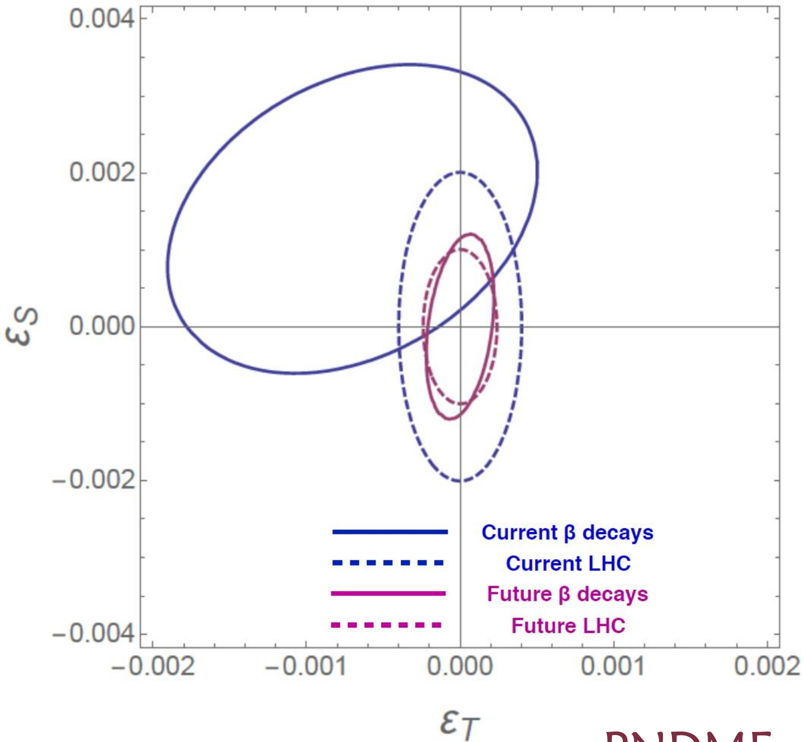
$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Low-Energy Expt → $O_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$



Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$|B_1 - b|_{\text{BSM}} < 10^{-3}$

$|b|_{\text{BSM}} < 10^{-3}$

CENPA: ^{6}He(b_{GT}) at 10^{-3}

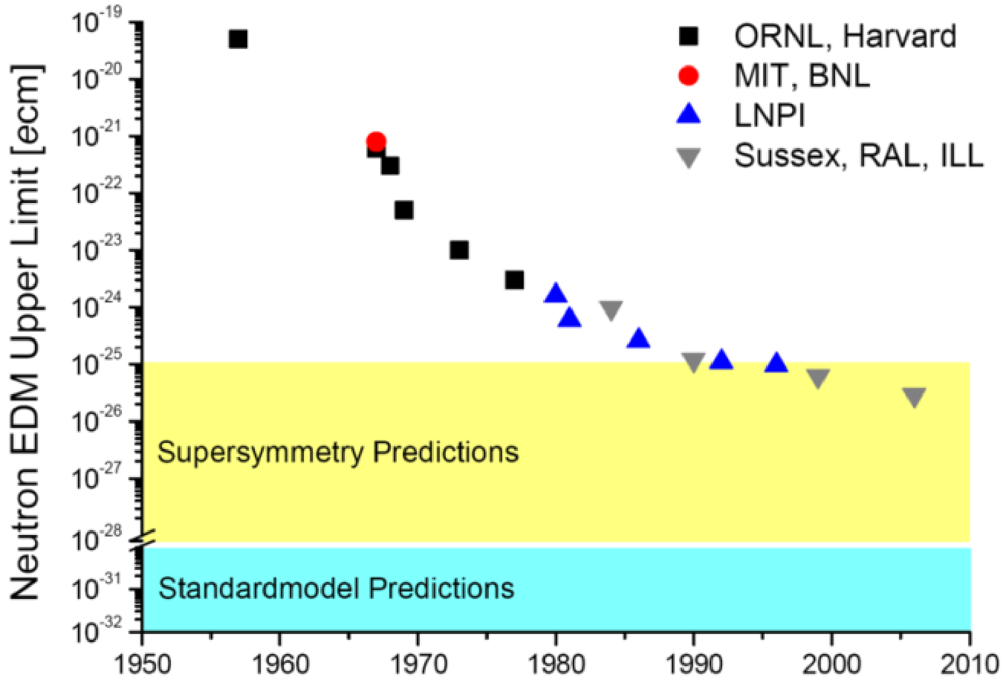
Plots by Vincenzo Cirigliano

PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049; 1806.09006

Electric Dipole Moment

§ Why do we care?

- CP-violating effect ⇒ Key ingredient for baryogenesis
  ⇒ Why matter exists
- Extremely small in SM: ≈ 10^{-31} e-cm (expect to probe 10^{-28} soon)
- Good candidate to constrain BSM models



Electric Dipole Moment

§ Why do we care?

- CP-violating effect ⇒ Key ingredient for baryogenesis
  ⇒ Why matter exists
- Extremely small in SM: ≈ 10^{-31} e-cm (expect to probe 10^{-28} soon)
- Good candidate to constrain BSM models

§ Lattice community are working on various contributions

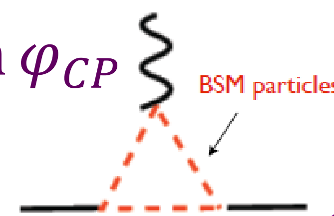
§ Lagrangian

$$L = L_{\mathrm{QCD}}^{CP\ \mathrm{Even}} + L_{\Theta} + L_{\mathrm{quark}}^{\mathrm{dim}-5} + L_{\mathrm{chromo-quark}}^{\mathrm{dim}-5} + \cdots$$

- If experiment sees signal before SM background
  ⇒ new physics
  ⇒ quark EDM (our focus here)

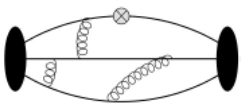
Induced by a variety of BSM scenarios

$d_i \propto \frac{m_i}{\Lambda^2} \sin \varphi_{CP}$



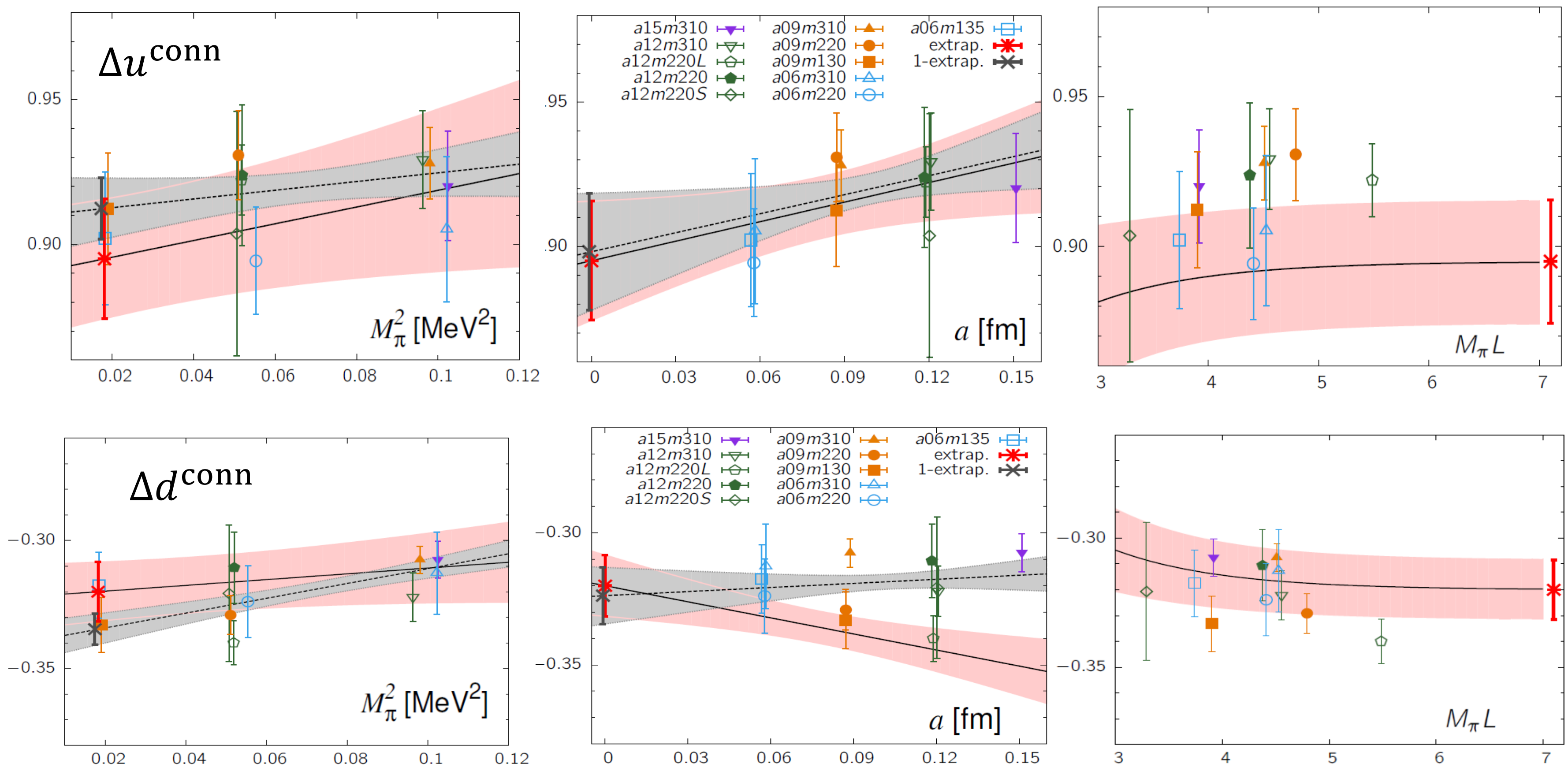
Quark Spin

§ Up and down quark "connected" contribution



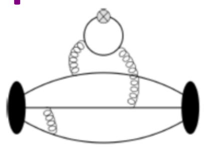
PNDME, 1806.09006, 1806.10604

$$\Delta q(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



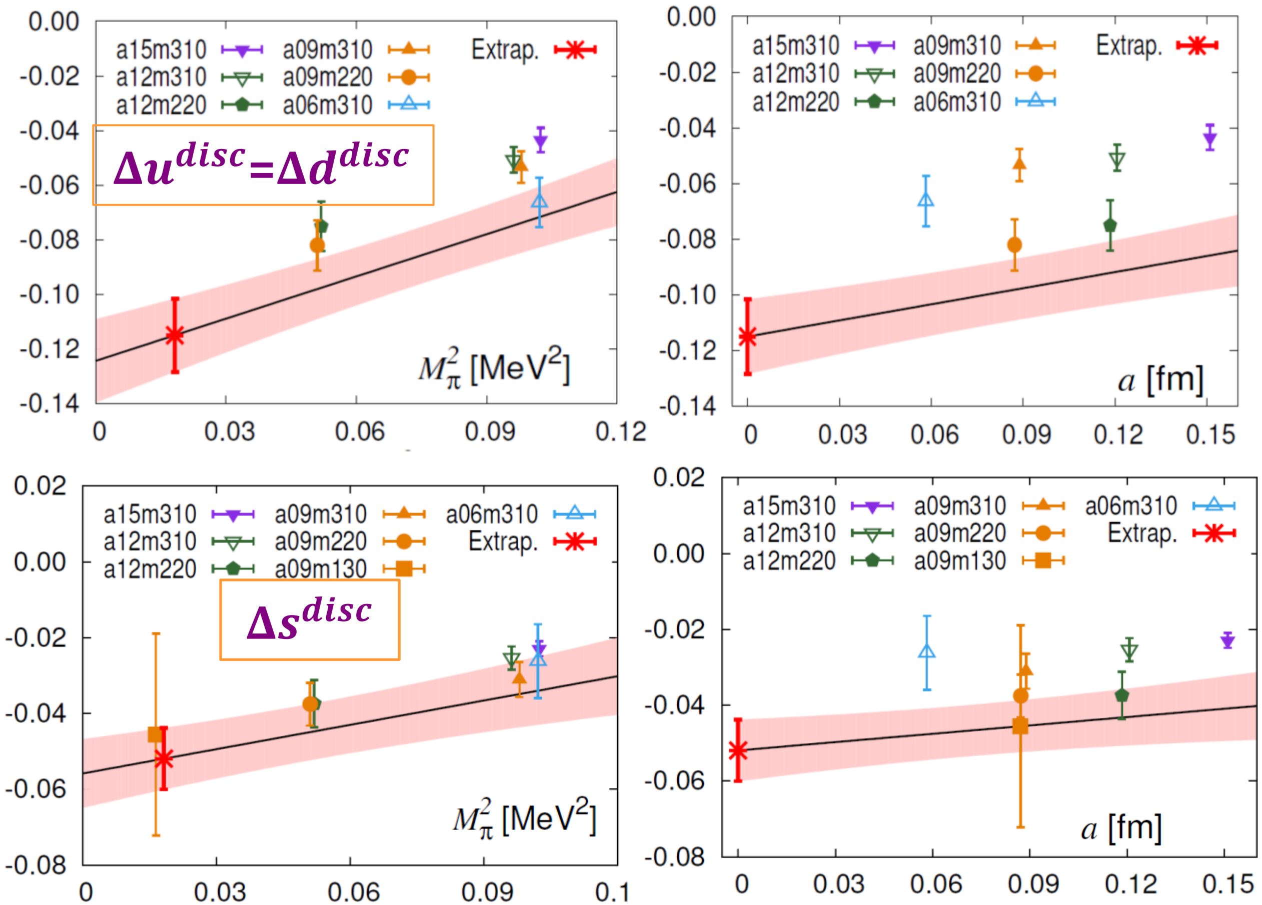
Quark Spin

§ Up and down quark “disconnected” contribution



PNDME, 1806.09006, 1806.10604

$$\Delta q^{\text{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



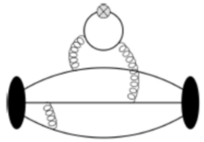
Anticipated pion-mass dependence

Unexpectedly strong lattice-spacing dependence!

Calculation at $a \approx$ 0.09 fm can have 50% change in Δu^{disc}

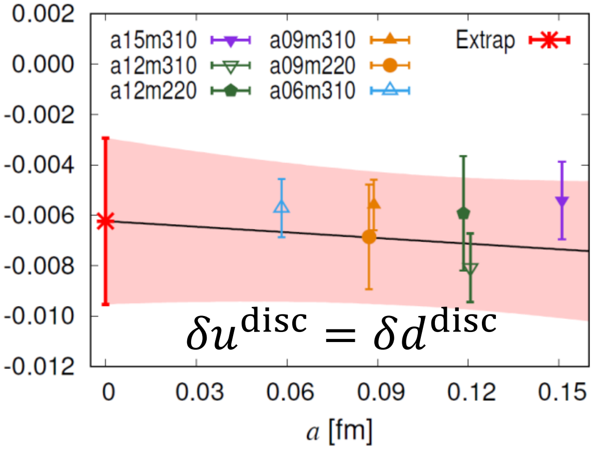
u/d/s Tensor Charges

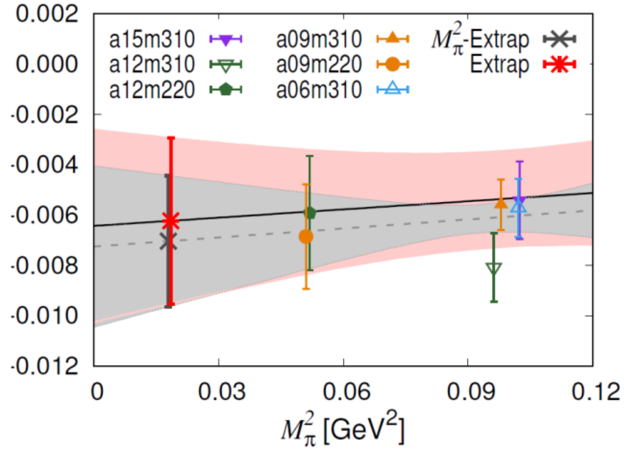
§ Up and down quark "disconnected" contribution



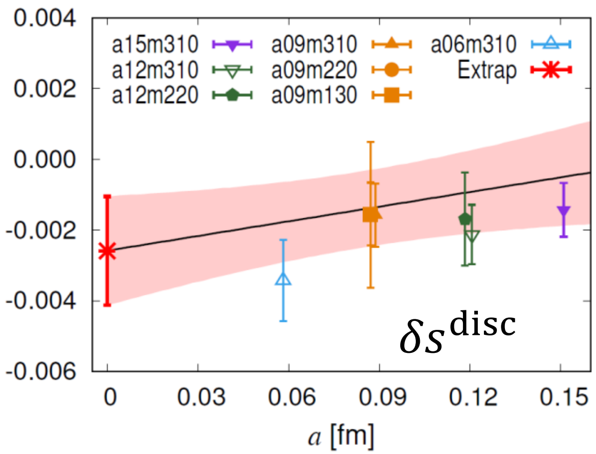
PNDME, 1806.09006, 1808.07597

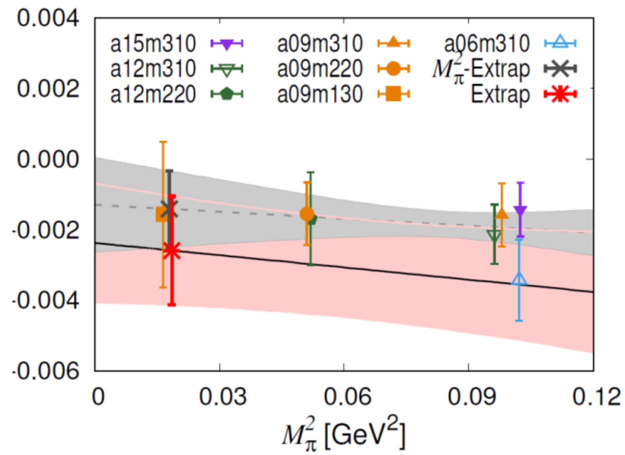
$$\delta q^{\text{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$





First time in LQCD



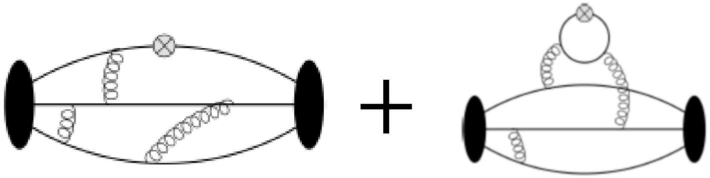


Mild dependence on a and pion mass!

Yong-Chull Jang

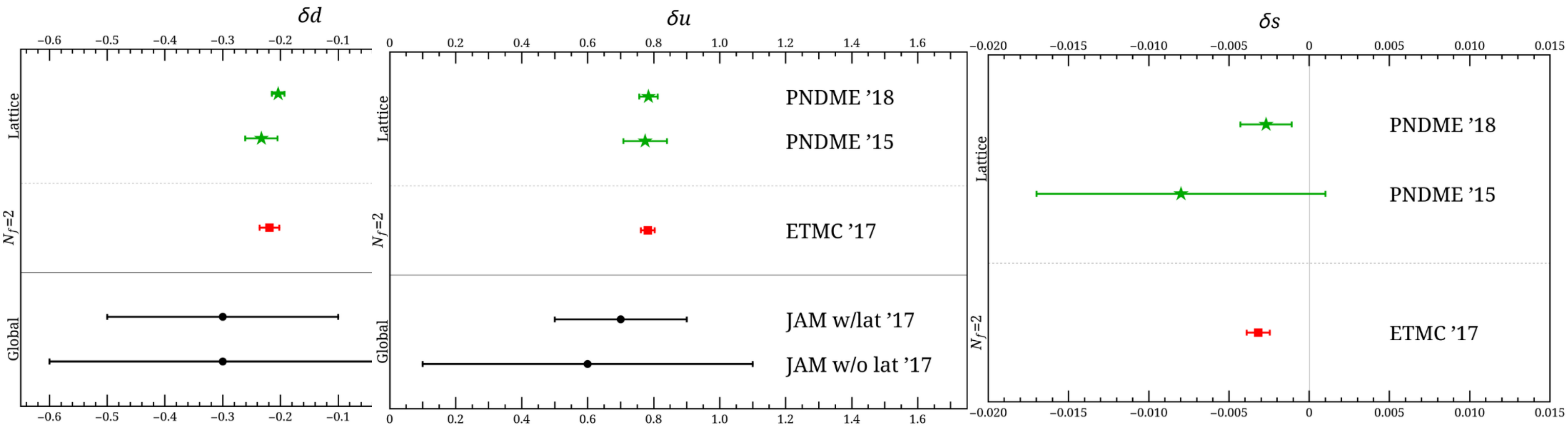
u/d/s Tensor Charges

§ Sum up both contributions



	g_T^u	g_T^d	g_T^s
Connected	0.790(27)	−0.198(10)	
Disconnected	−0.0064(33)	−0.0064(33)	−0.0027(16)
PDNME'18 (Sum)	0.784(28)	−0.204(11)	−0.0027(16)
ETMC'17 [14]	0.782(21)	−0.219(17)	−0.00319(72)
PNDME'15 [5]	0.774(66)	−0.233(28)	0.008(9)

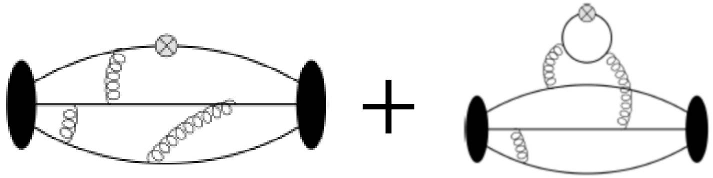
Calculation from one lattice ensemble only
No cont. extrapolation errors



PNDME, 1806.09006, 1808.07597

u/d/s Tensor Charges

§ Sum up both contributions



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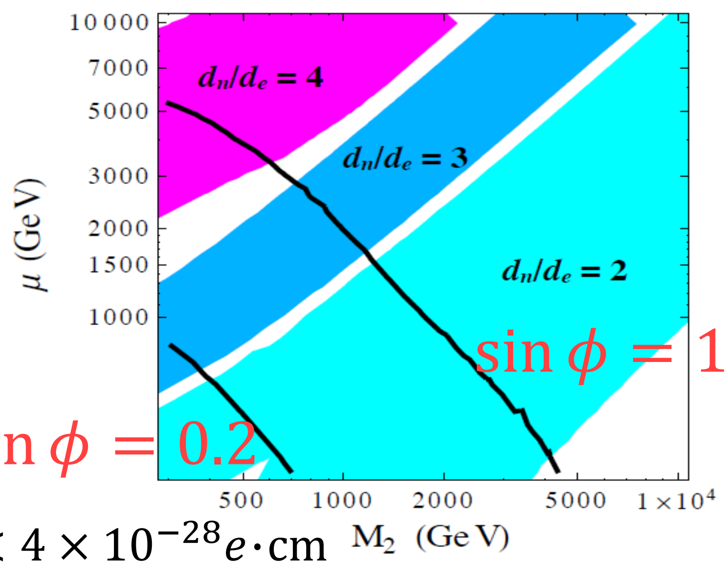
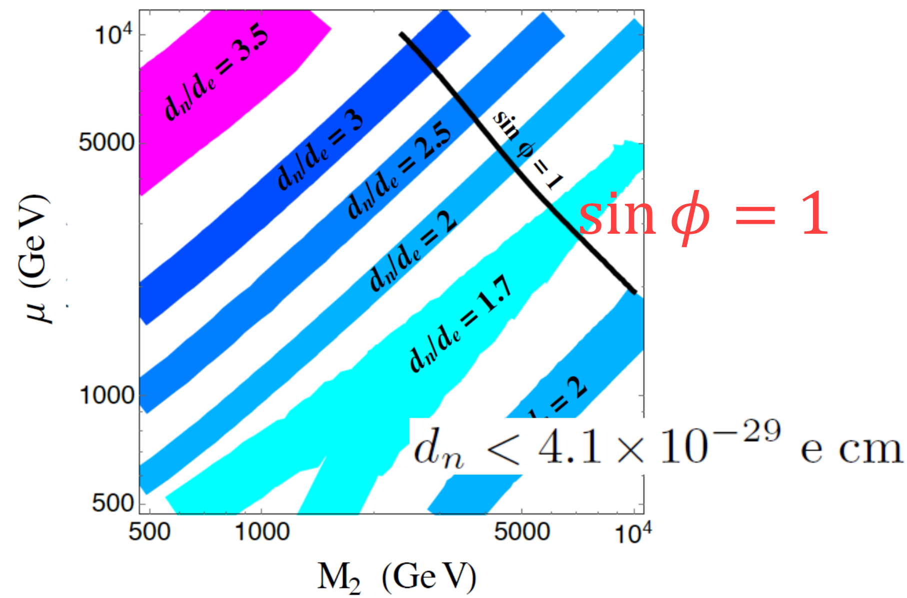
Calculation from one lattice ensemble only
No cont. extrapolation errors

Set limits for nEDM in split-SUSY scenario with gaugino mass unification



$d_n < 4 \times 10^{-28} e\cdot$cm

PNDME, 1506.04196; 1506.06411



PNDME, 1806.09006, 1808.07597

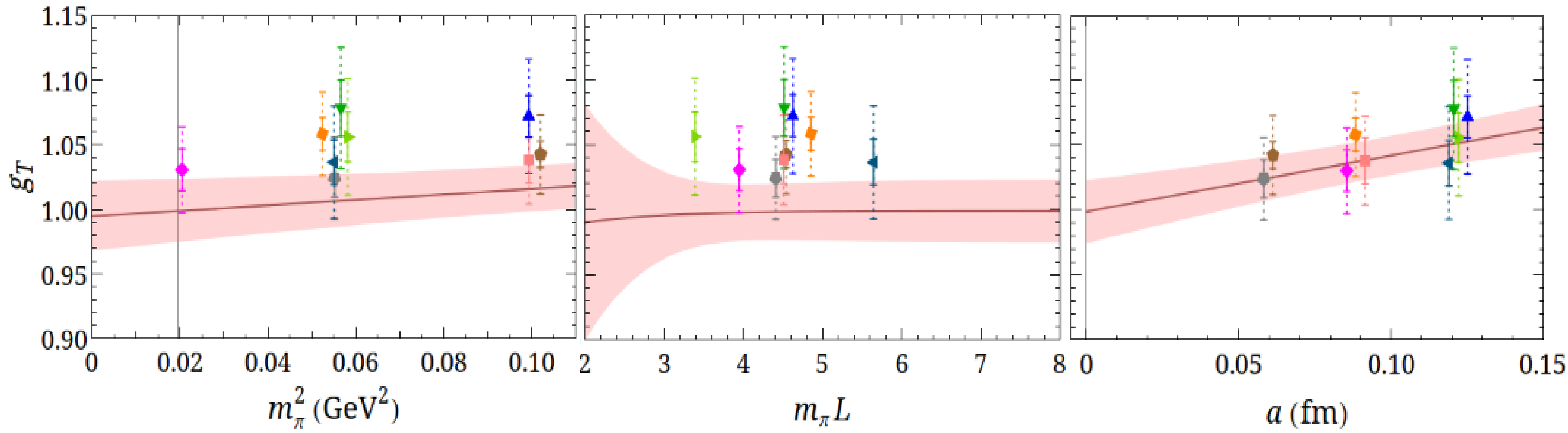
Precision Nucleon Couplings

- § g_T: zeroth moment of transversity $\Gamma = \sigma_{\mu\nu}$
- § A state-of-the art calculation (PNDME)

$$g_T = \int_{-1}^{1} dx\, \delta q(x)$$

➢ Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

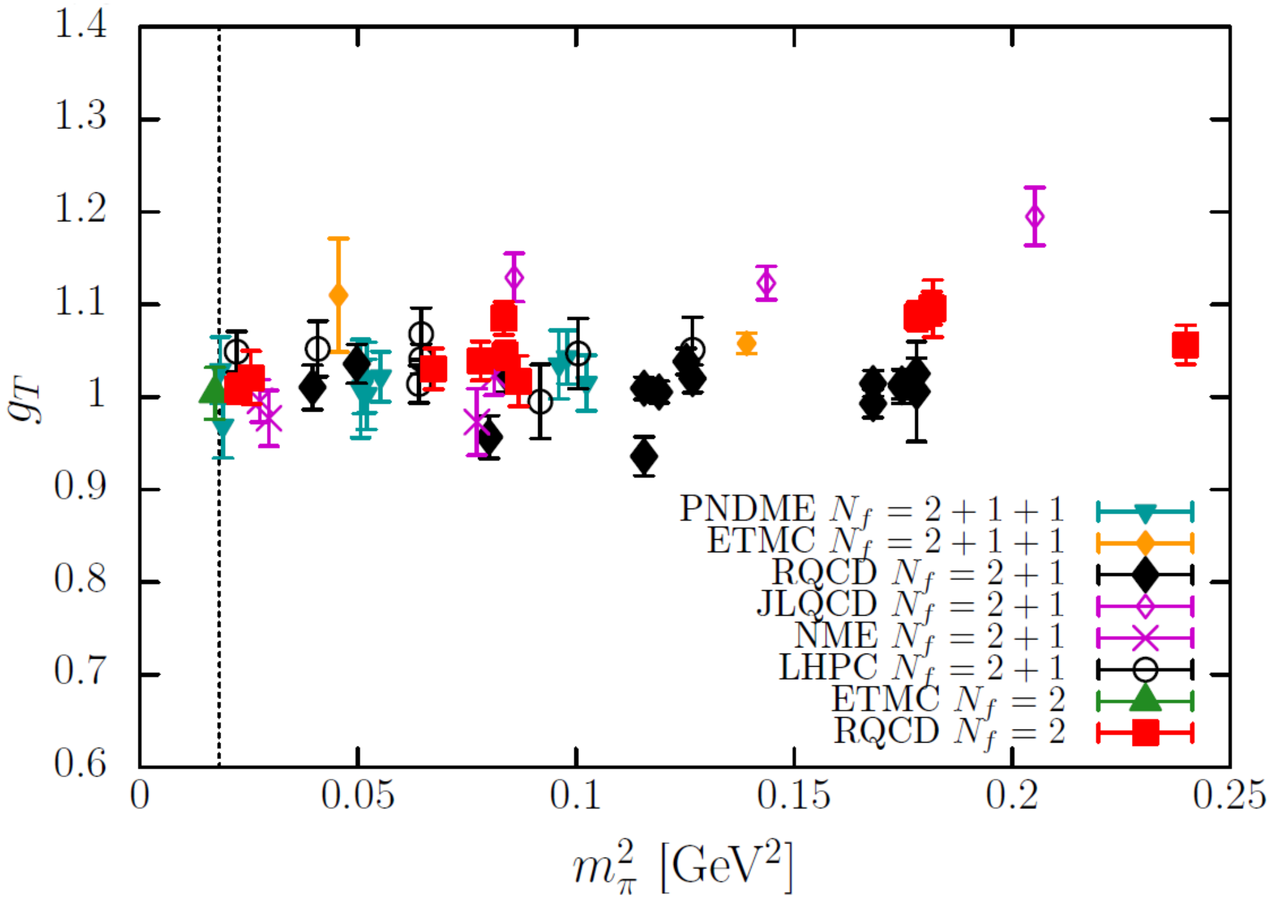


First extrapolation to the physical limit
of a nucleon matrix element!

Precision Nucleon Couplings

§ Usually more than one LQCD calculation

- For example, tensor charge
- Lattice results should agree in the continuum limit



Plot by Sara Collins @ Lattice 2016

Precision Nucleon Couplings

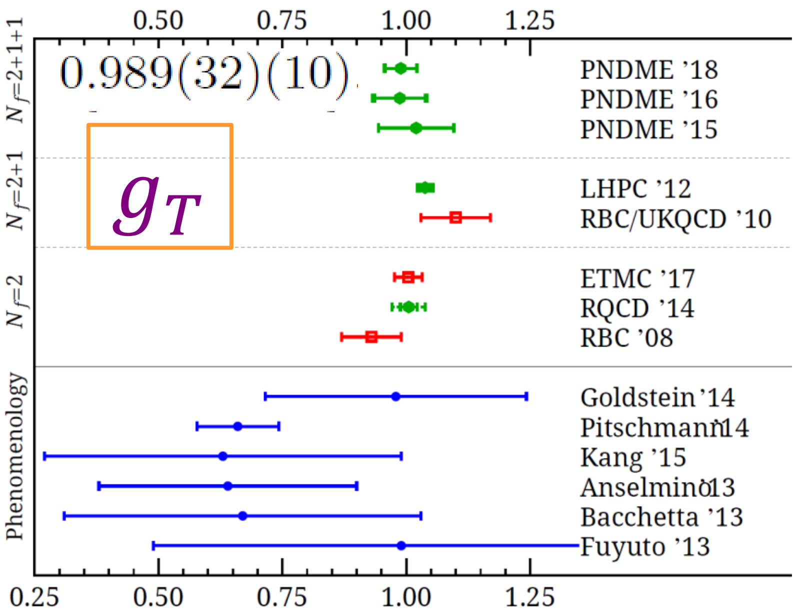
FLAG rating system

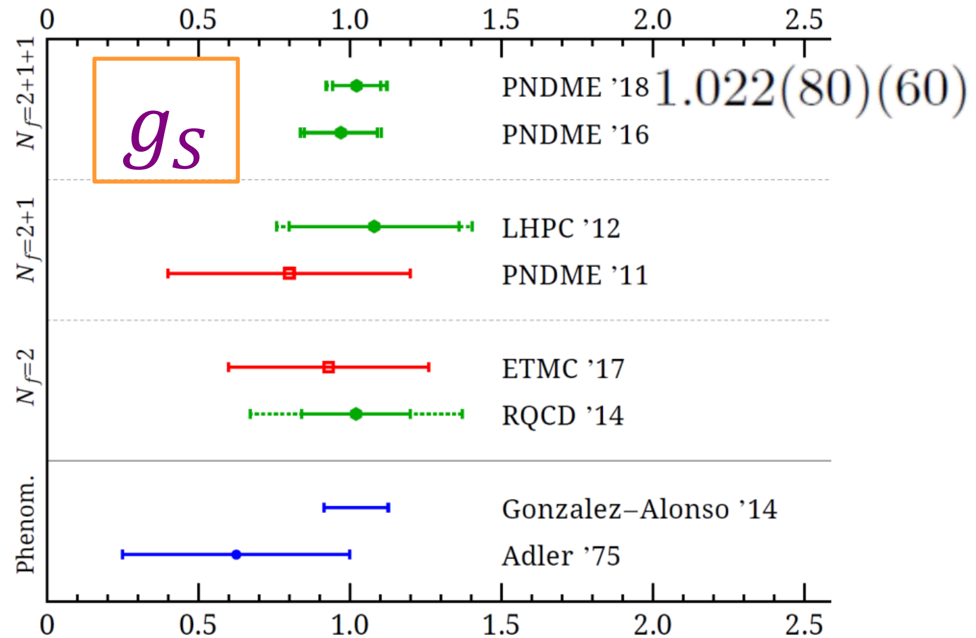
PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

PNDME, 1806.09006



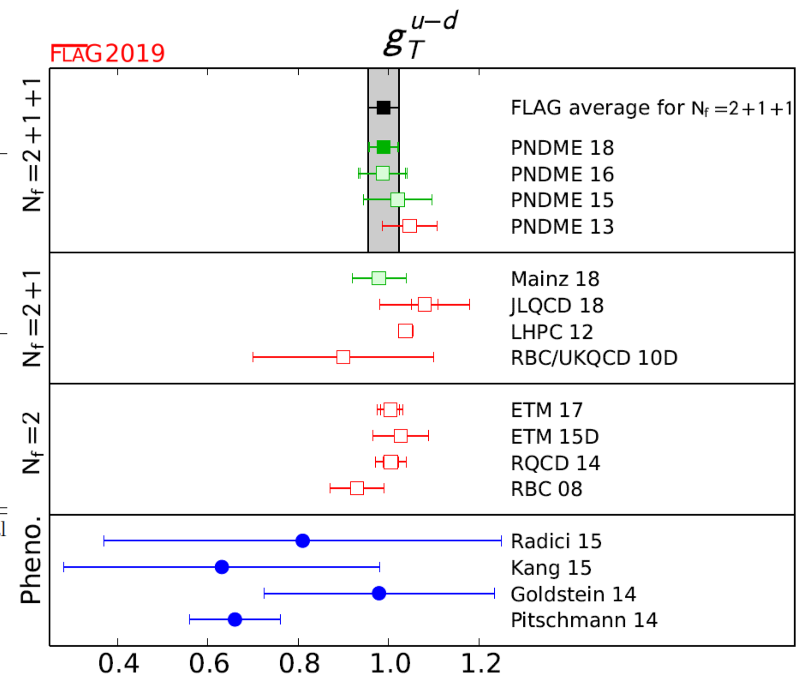


FLAG 2019

§ Finally adopted by FLAG! https://arxiv.org/pdf/1902.08191.pdf

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_T^{u-d}
PNDME 18	[84]	2+1+1	A	★‡	★	★	★	★	0.989(32)(10)
PNDME 16	[830]	2+1+1	A	○‡	★	★	★	★	0.987(51)(20)
PNDME 15	[828, 829]	2+1+1	A	○‡	★	★	★	★	1.020(76)
PNDME 13	[827]	2+1+1	A	■‡	■	★	★	★	1.047(61)
Mainz 18	[915]	2+1	C	★	○	★	★	★	0.979(60)
JLQCD 18	[839]	2+1	A	■	○	○	★	★	1.08(3)(3)(9)
LHPC 12	[920]	2+1	A	■‡	★	★	★	★	1.038(11)(12)
RBC/UKQCD 10D	[834]	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	[826]	2	A	■	○	○	★	★	1.004(21)(2)(19)
ETM 15D	[822]	2	A	■	○	○	★	★	1.027(62)
RQCD 14	[819]	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	[918]	2	A	■	■	■	★	■	0.93(6)

‡ The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.



Total Quark Intrinsic Spin

Not "equal": systematics are different?

- PNDME: 0.143(31)(36) (2+1+1 flavor clover-on-HISQ)
- ETMC: 0.201(17)(5) (2 flavor twisted mass)
- χQCD: 0.202(13)(19) (2+1 flavor overlap-on-Domain Wall)

	g_A^{u-d}	$a \to 0$	M_π MeV	$M_\pi L$	Z_A
PNDME N_f =2+1+1	1.218(25)(30)	Yes 11 ensembles 0.15 – 0.06 fm	135 220 310	3.3 – 5.5	Assume $Z_A^s = Z_A^{ns}$
ETMC N_f =2	1.212(40)	0.094 fm	130	2.93	Checked $Z_A^s = Z_A^{ns}$
χQCD N_f =2+1	1.254(16)(30)	"No" a variation 0.143 fm 0.11 fm 0.083 fm	 171 337 302	 3.97 4.53 4.06	Checked $Z_A^s = Z_A^{ns}$

In perturbation theory $Z_A^s \neq Z_A^{ns}$ at 2 loops . ETMC & χQCD show a ~1% difference

Slide from Rajan Gupta @ Spin 2018

Proton Mass Decomposition

$$M = -\langle T_{44}\rangle = \langle H_m\rangle + \langle H_E\rangle(\mu) + \langle H_g\rangle(\mu) + \frac{1}{4}\langle H_a\rangle$$

$$M = -\langle T_{\mu\mu}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

X. Ji, PRL74:1071 (1995)

quark mass: $\langle H_m\rangle = \sum_{u,d,s,\ldots}\int d^3x\, m\bar{\psi}\psi$

quark energy: $\langle H_E\rangle = \frac{3}{4}\left(\langle x\rangle_q M - \langle H_m\rangle\right)$

glue energy: $\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M$

anomaly: $\langle H_a\rangle = \langle H_g^a\rangle + \langle H_m^\gamma\rangle$

$$\langle x\rangle_{q,g} = \int_0^1 dx\, x f_{q,g}(x) = -\frac{\langle N|\frac{4}{3}\bar{T}_{44}^{q,g}|N\rangle}{M\langle N|N\rangle}$$

$$\bar{T}_{44}^q = \int d^3x\, \bar{\psi}\frac{1}{2}\left(\gamma_4\overleftrightarrow{D}_4 - \frac{1}{4}\sum_{i=0,1,2,3}\gamma_i\overleftrightarrow{D}_i\right)\psi$$

$$\bar{T}_{44}^g = \int d^3x\, \frac{1}{2}(E^2 - B^2)$$

$$\langle H_g^a\rangle = \int d^3x\, \frac{-\beta(g)}{g}(E^2 + B^2)$$

$$\langle H_m^\gamma\rangle = \sum_{u,d,s,\ldots}\int d^3x\, \gamma_m m\bar{\psi}\psi$$

Ingredients

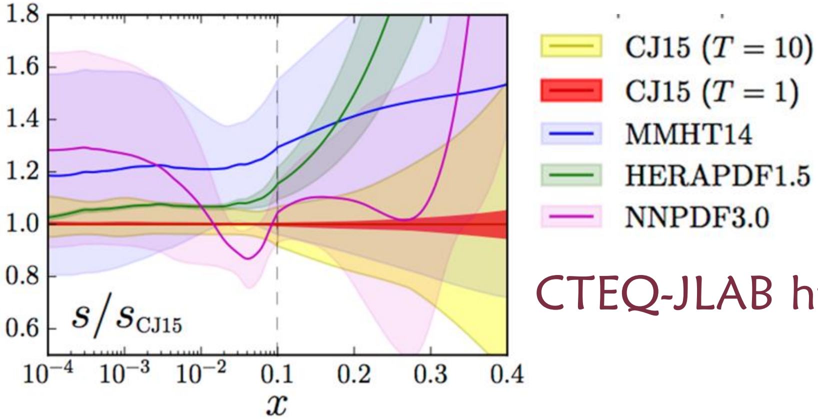
- proton mass
- scalar charge
- momentum fractions (both quark and glue)
- renormalization of momentum fractions including mixing

x

Slide from Keh-Fei Liu

First Lattice Strange PDF

§ Large uncertainties in global PDFs



- Assumptions imposed due to lack of precision data
  $s = \bar{s} = \kappa(\bar{u} + \bar{d})$

CTEQ-JLAB https://www.jlab.org/theory/cj/

First Lattice Strange PDF

§ Large uncertainties in global PDFs

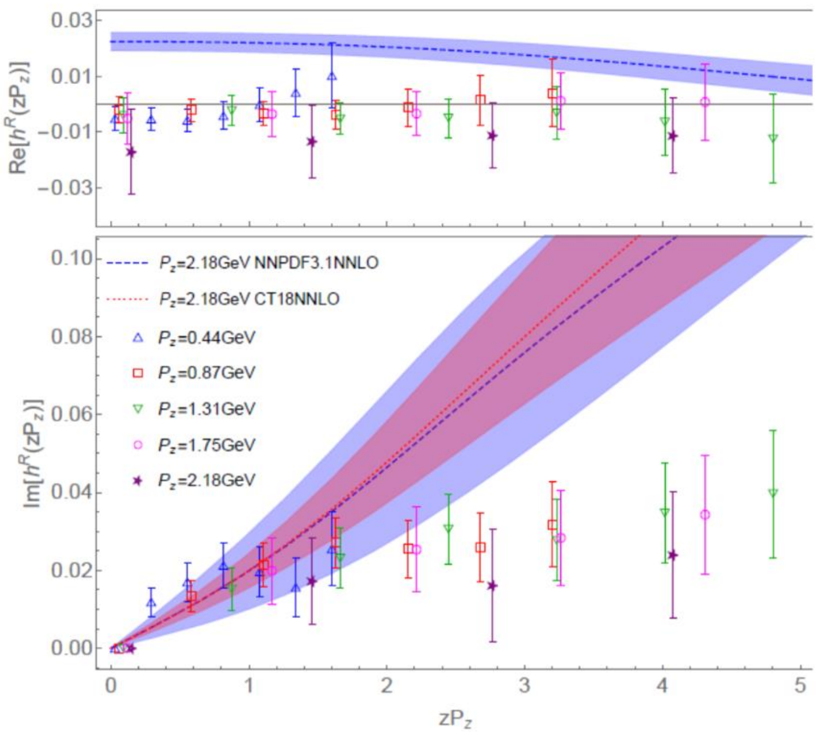
$\mathrm{Re}[h(z)] \propto$

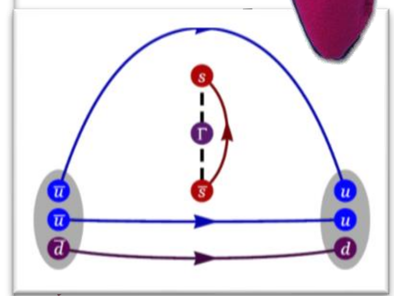
$$\int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$

$\mathrm{Im}[h(z)] \propto$

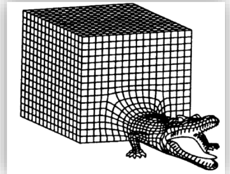
$$\int dx\,(s(x) + \bar{s}(x))\sin(xzP_z)$$

- symmetric $s - \bar{s}$ distribution.
- smaller momentum fraction





G: Rui Zhang



Slide by Rui Zhang @ DNP 2020

Gluon Helicity

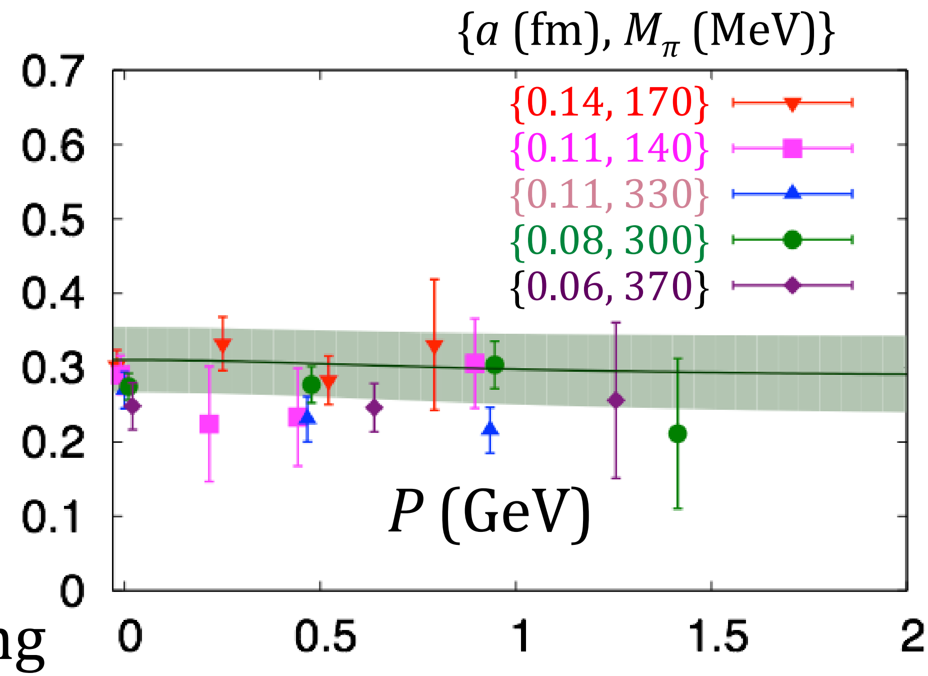
- § Jaffe & Manohar, 1990 $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q^z + \mathcal{L}_g^z$
- § Can be calculated through large-momentum frame

X. Ji et al., PRL. 111 (2013) 112002; 110 (2013) 262002; PRD 89, 085030 (2014)

$$S_G(P)\, S_z = \frac{\left\langle PS \middle| \int d^3x\, (\vec{E} \times \vec{A}_{\text{phys}})_z \middle| PS \right\rangle}{2E_P}$$

- § First results by **χQCD**

$$\Delta G(\mu^2 = 10\ \text{GeV}^2) \approx S_G(\infty, \mu^2 = 10\ \text{GeV}^2) = 0.287(55)(16)$$



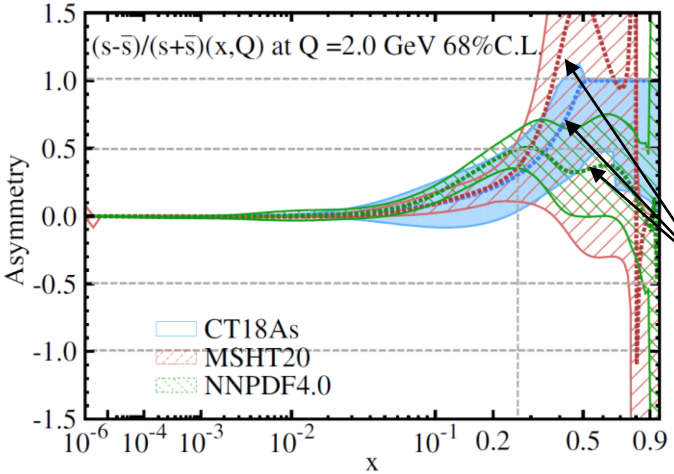
Yang et al, Phys. Rev. Lett. 118 (2017) 102001

- Future improvement to matching

- § Current limit
- DSSV14 $\int_{0.05}^{1} dx\, \Delta G(10^2\ \text{GeV}, x) \approx [0.14, 0.24]$

Lattice Impact on Strange PDF

- lattice QCD can constrain PDFs (polarized, meson, TMDs, GPDs,…) that are difficult to access in experiments
- Example: the strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure (left), but can be predicted in lattice QCD (right)



differences reflect the pulls of LHC and other experiments

Lattice Impact on Strange PDF

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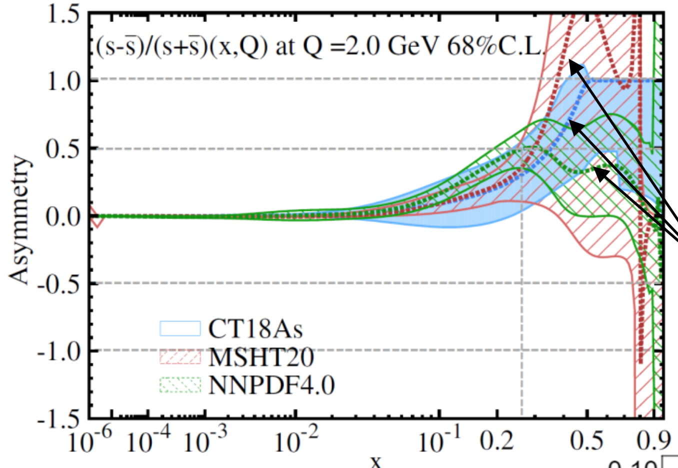
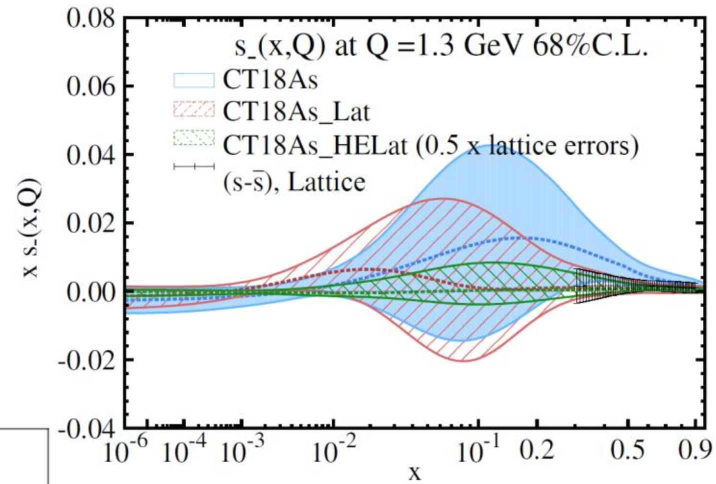
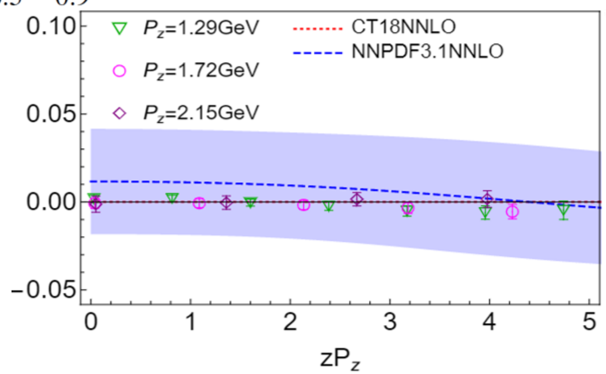


differences reflect the pulls of LHC and other experiments



$\text{Re}[h(z)] \propto$
$\int dx\, \left(s(x) - \bar{s}(x)\right) \cos(xzP_z)$



T. Hou, HL, M. Yan, C.-P. Yuan, 2204.07944