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The Nuclear Force through the Decades

Charlotte Elster

Supported by U.S. DEPARTMENT OF ENERGY Office of Science

Department of physics + astronomy

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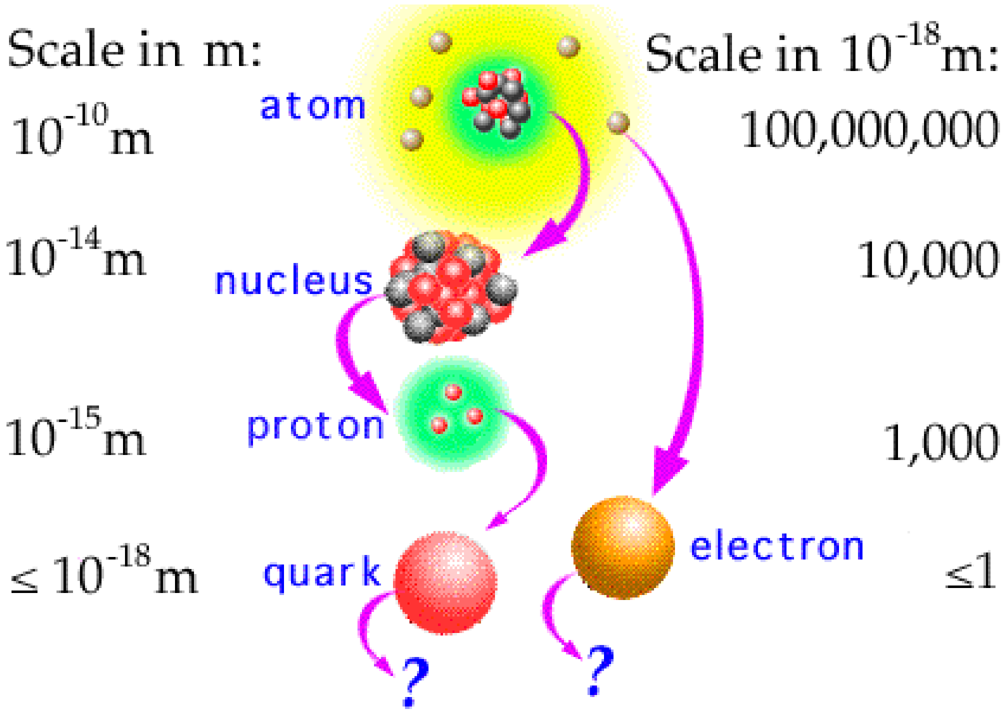
Scales in Nature

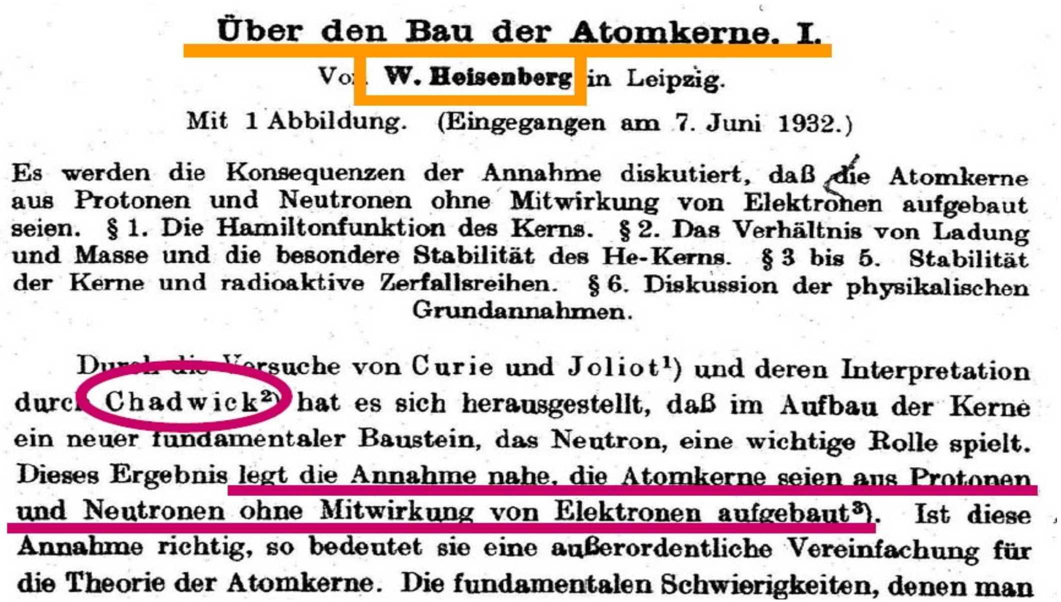


Early History of Nuclear Physics

1932

- Chadwick: Discovery of the neutron
- Heisenberg: Postulates Iso-Spin

… suggests the assumption that atomic nuclei are built from protons and neutrons without electrons ..



Über den Bau der Atomkerne. I.

Von **W. Heisenberg** in Leipzig.

Mit 1 Abbildung. (Eingegangen am 7. Juni 1932.)

Es werden die Konsequenzen der Annahme diskutiert, daß die Atomkerne aus Protonen und Neutronen ohne Mitwirkung von Elektronen aufgebaut seien. § 1. Die Hamiltonfunktion des Kerns. § 2. Das Verhältnis von Ladung und Masse und die besondere Stabilität des He-Kerns. § 3 bis 5. Stabilität der Kerne und radioaktive Zerfallsreihen. § 6. Diskussion der physikalischen Grundannahmen.

Durch die Versuche von Curie und Joliot[1]) und deren Interpretation durch Chadwick[2]) hat es sich herausgestellt, daß im Aufbau der Kerne ein neuer fundamentaler Baustein, das Neutron, eine wichtige Rolle spielt. Dieses Ergebnis legt die Annahme nahe, die Atomkerne seien aus Protonen und Neutronen ohne Mitwirkung von Elektronen aufgebaut[3]). Ist diese Annahme richtig, so bedeutet sie eine außerordentliche Vereinfachung für die Theorie der Atomkerne. Die fundamentalen Schwierigkeiten, denen man

Concept of Iso-Spin (Isobaric Spin)

- The mass of the neutron and the proton are almost identical: they are nearly degenerate, and both are thus often called **nucleon**
- Although the proton has a positive charge, and the neutron is neutral, they are almost identical in all other respects.
- The strong interaction between any pair of nucleons is almost the same, independent of whether they are interacting as protons or as neutrons
- Iso-spin operator τ in iso-spin space: proton: $\tau_z = 1/2$ neutron: $\tau_z = -1/2$
- SU(2) symmetry: follows the same algebra as spin
- Can be considered as rotation in iso-spin space
- Neutron and proton characterized by projections of τ

Early History of Nuclear Physics

1932 Chadwick: Discovery of the neutron
Heisenberg: Postulates Iso-Spin

1935 Yukawa: Meson Hypothesis

From:
H. Yukawa,
Proc. Phys.Math.Soc. Japan **17**, 48 (1935)

Nuclear Force is mediated by the exchange of a particle with mass

§ 2. Field describing the interaction

In analogy with the scalar potential of the electromagnetic field, a function $U(x, y, z, t)$ is introducd to describe the field between the neutron and the proton. This function will satisfy an equation similar to the wave equation for the electromagnetic potential.

Now the eqnation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\} U = 0 \qquad (1)$$

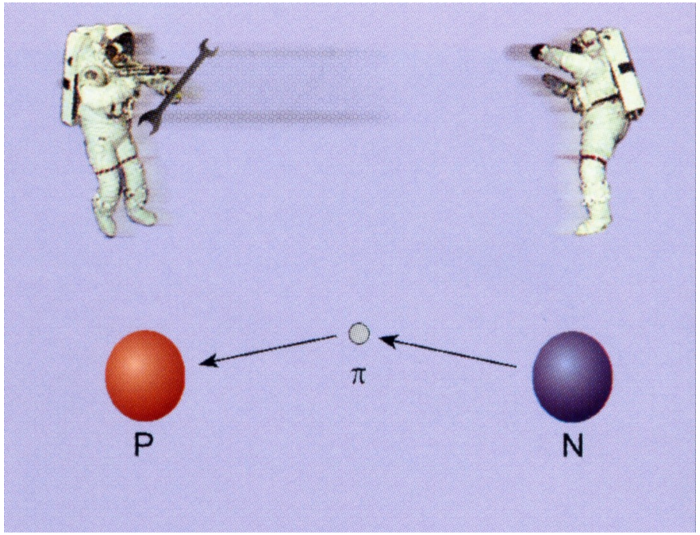
has only static solution with central symmetry $\frac{1}{r}$, except the additive and the multiplicative constants. The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2\frac{e^{-\lambda r}}{r}, \qquad (2)$$

Repulsive force mediated through exchange of particle (here wrench)

Yukawa prediction:

Mass $\approx$ 140 MeV



Birth of
Particle Physics

Attractive force: think of Boomerang exchanged

Early History of Nuclear Physics

1932

- Chadwick: Discovery of the neutron
- Heisenberg: Postulates Iso-Spin

1935

Yukawa: Meson Hypothesis

1940

Discovery of the pion in cosmic rays (1947) and in the Berkeley Cyclotron Lab (1948)

Yukawa: Nobelprize (1949)

1950

One-Pion-Exchange (OPE) o.k.

Taketani, Nakamura, Sasaki (1951) 3 ranges

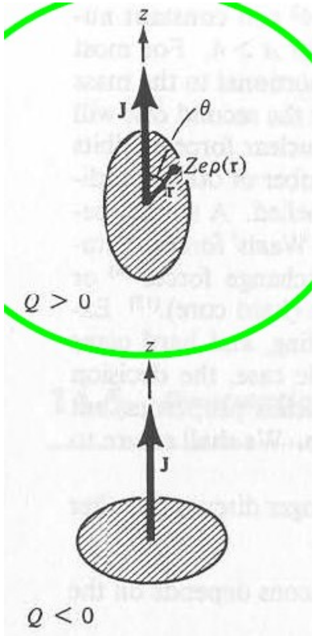
Multi-pion exchange theories –

BUT - Problems with renormalization

Information about the nuclear force

Binding energy	2.225 MeV
Spin, parity	1^+
Isospin	0
Magnetic moment	μ=0.857 μ_N
Electric quadrupole moment	Q=0.282 e fm^2

Deuteron

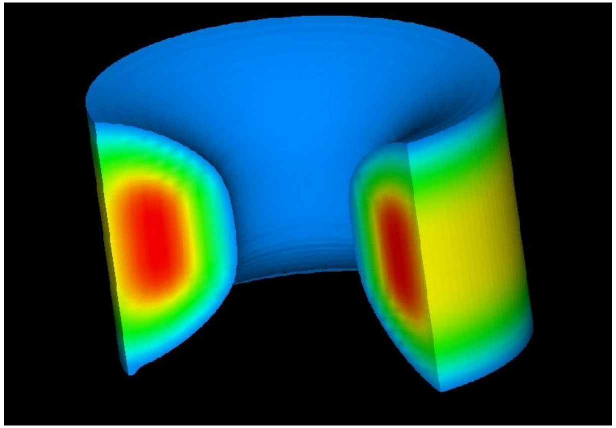


$$\mu_p + \mu_n = 2.792\mu_N - 1.913\mu_N = 0.879\mu_N$$

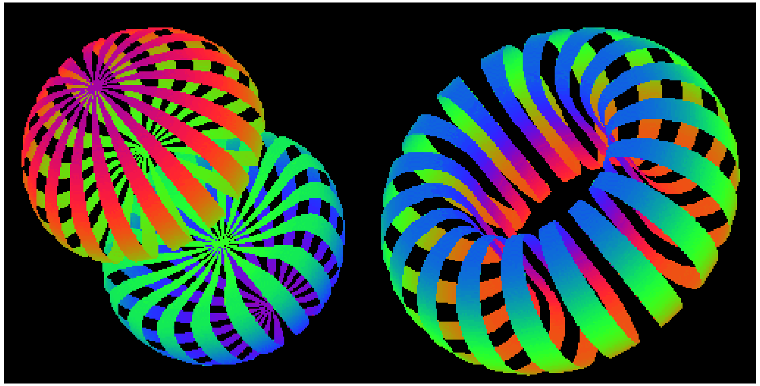
$$|\psi_d\rangle = 0.98|^3S_1\rangle + 0.20|^3D_1\rangle$$

rms radius=1.963 fm

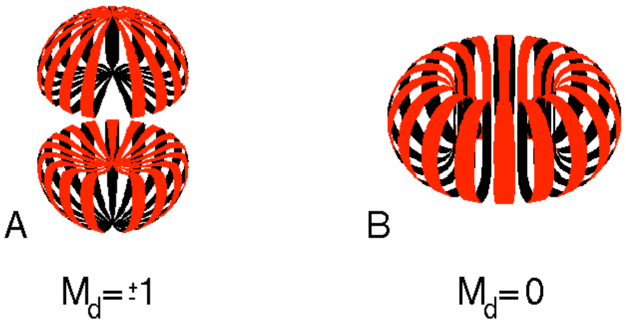
Deuteron is not spherical



Probability density for both
Nucleons having spin down



Deuteron shapes from AV18



Tensor Force

$$(-S_{12}) = -3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

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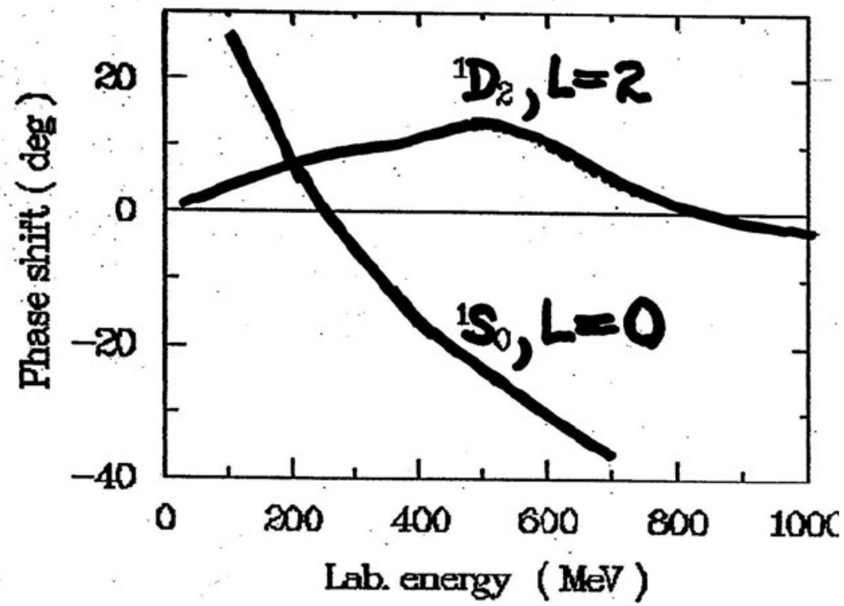
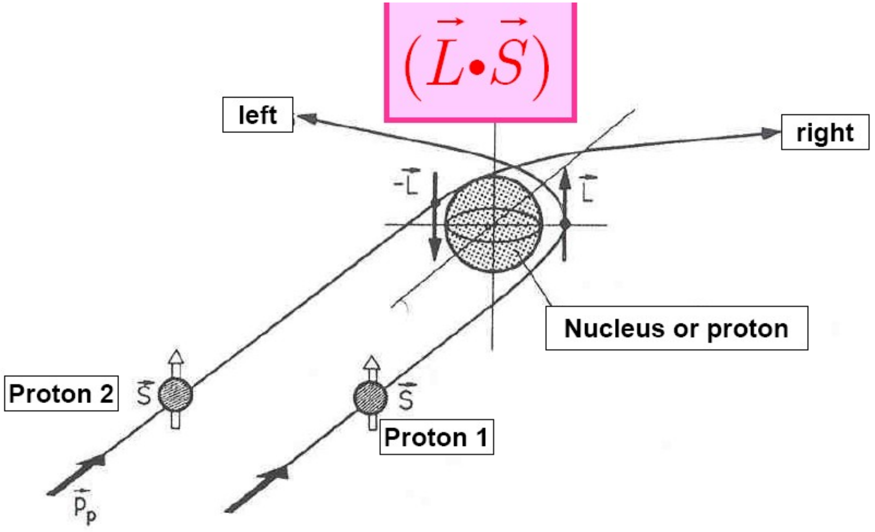
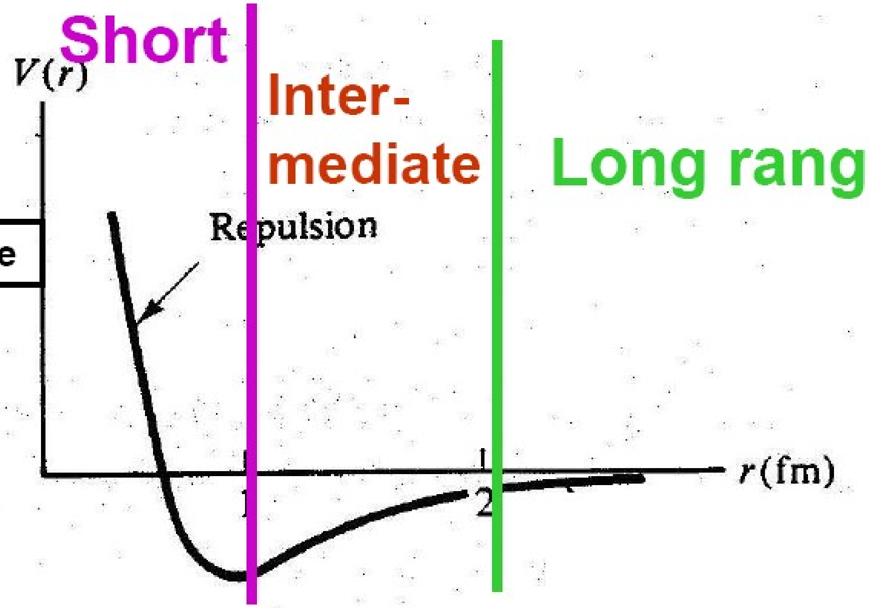
Spin Orbit Force





General structure of the nuclear force



Central force

Tensor force

Spin-orbit force

Most General Form of the NN Force: Galilei Invariance

ANNALS OF PHYSICS: 4, 166–179 (1958)

Velocity Dependence of the Two-Nucleon Interaction*

S. OKUBO AND R. E. MARSHAK

University of Rochester, Rochester, New York

Invariance arguments are used to derive the most general velocity dependent charge independent two-nucleon interaction in the nonrelativistic approximation. If one stays on the energy shell, the only essentially new term in the two-nucleon interaction is the quadratic spin-orbit potential. Off the energy shell, the quadratic spin-momentum potential must be treated as independent. In meson theory, the quadratic terms arise as $(\mu/M)^2$ (μ is the pion mass, M the nucleon mass) corrections to the second-order static potential in contrast to the linear spin-orbit potential which originates as a (μ/M) correction to the fourth-order static potential.

Two spin-1/2 particles:
at most 5 linear independent spin-momentum operators

Vectors in R^3:
p´-p   p´+p and p´ x p

$$
\begin{aligned}
w_1(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= 1 \\
w_2(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
w_3(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= i\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathrm{p} \times \mathrm{p}') \\
w_4(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathrm{p} \times \mathrm{p}')\, \boldsymbol{\sigma}_2 \cdot (\mathrm{p} \times \mathrm{p}') \\
w_5(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathrm{p}' + \mathrm{p})\, \boldsymbol{\sigma}_2 \cdot (\mathrm{p}' + \mathrm{p}) \\
w_6(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}', \mathrm{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathrm{p}' - \mathrm{p})\, \boldsymbol{\sigma}_2 \cdot (\mathrm{p}' - \mathrm{p})
\end{aligned}
$$

on-shell:

$$
\begin{aligned}
\sigma_1 \cdot \sigma_2 = & \frac{1}{(\mathbf{p} \times \mathbf{p}')^2} \sigma_1 \cdot (\mathbf{p} \times \mathbf{p}') \sigma_2 \cdot (\mathbf{p} \times \mathbf{p}') \\
& + \frac{1}{(\mathbf{p} + \mathbf{p}')^2} \sigma_1 \cdot (\mathbf{p} + \mathbf{p}') \sigma_2 \cdot (\mathbf{p} + \mathbf{p}') \\
& + \frac{1}{(\mathbf{p} - \mathbf{p}')^2} \sigma_1 \cdot (\mathbf{p} - \mathbf{p}') \sigma_2 \cdot (\mathbf{p} - \mathbf{p}').
\end{aligned}
$$

⇒ **5 linear independent amplitudes**

Wolfenstein Amplitudes

59. Wolfenstein L, Ashkin J. Invariance conditions on the scattering amplitudes for spin ½ particles. *Phys Rev* (1952) 85:947–9. doi:10.1103/physrev.85.947

2010

PHYSICAL REVIEW C **81**, 034006 (2010)

Two-nucleon systems in three dimensions

J. Golak,[1] W. Glöckle,[2] R. Skibiński,[1] H. Witała,[1] D. Rozpędzik,[1] K. Topolnicki,[1] I. Fachruddin,[3] Ch. Elster,[4] and A. Nogga[5]

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[2]*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[3]*Departemen Fisika, Universitas Indonesia, Depok 16424, Indonesia*

[4]*Institute of Nuclear and Particle Physics, Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA*

[5]*Forschungszentrum Jülich, Institut für Kernphysik (Theorie), Institute for Advanced Simulation and Jülich Center for Hadron Physics, D-52425 Jülich, Germany*

(Received 11 January 2010; published 24 March 2010)

A recently developed formulation for treating two- and three-nucleon bound states in a three-dimensional formulation based on spin-momentum operators is extended to nucleon-nucleon scattering. Here the nucleon-nucleon T-matrix is represented by six spin-momentum operators accompanied by six scalar functions of momentum vectors. We present the formulation and provide numerical examples for the deuteron and nucleon-nucleon scattering observables. A comparison to results from a standard partial-wave decomposition establishes the reliability of this formulation.

DOI: 10.1103/PhysRevC.81.034006 PACS number(s): 21.30.−x, 21.45.Bc

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Operators from symmetry consideration

- **Spin-momentum**

1	central
$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	Spin-spin
$i\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathrm{p} \times \mathrm{p}')$	Spin-orbit
$\boldsymbol{\sigma}_1 \cdot (\mathrm{p} \times \mathrm{p}')\ \boldsymbol{\sigma}_2 \cdot (\mathrm{p} \times \mathrm{p}')$	Quadratic spin-orbit
$\boldsymbol{\sigma}_1 \cdot (\mathrm{p}' + \mathrm{p})\ \boldsymbol{\sigma}_2 \cdot (\mathrm{p}' + \mathrm{p})$	
$\boldsymbol{\sigma}_1 \cdot (\mathrm{p}' - \mathrm{p})\ \boldsymbol{\sigma}_2 \cdot (\mathrm{p}' - \mathrm{p})$	Tensor

- **Spin-position**

Spin-orbit **L · S**

Tensor

$$(-S_{12}) = -3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Once Operator structure is fixed:
Each operator can be multiplied with scalar functions depending on magnitudes of momenta / positions

Phenomenology

Department of physics + astronomy

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First:

Calculation of Phenomenological Nucleon-Nucleon Potentials*

J. L. GAMMEL, R. S. CHRISTIAN, AND R. M. THALER
Los Alamos Scientific Laboratory, Los Alamos, New Mexico
(Received August 27, 1956)

An attempt to find a phenomenological nucleon-nucleon potential is described. The class of charge and velocity independent potentials with central and tensor parts of Yukawa shape with a hard core is considered. The depths, ranges, and core radii of such potentials with general spin and parity dependence are adjusted to fit experimental data. No potential of this type is found which fits all of the data.

II. FORM OF THE POTENTIALS

The potentials considered in this paper are all of the form

$$V(r)=\infty, \qquad r<r_0$$
$$V(r)=V_c(r)+V_t(r)S_{12}, \qquad r>r_0, \tag{1}$$

where S_{12} is the tensor operator.[8] The potentials are assumed to have a Yukawa shape; that is,

$$V_c(r)=-V_c\exp(-r/r_c)/(r/r_c),$$
$$V_t(r)=-V_t\exp(-r/r_t)/(r/r_t), \tag{2}$$
$$\mu_c\equiv 1/r_c, \quad \mu_t\equiv 1/r_t.$$

The five parameters V_c, r_c, V_t, r_t, and r_0 depend on the spin and parity ($V_t=0$ for $S=0$). We assume that the radius of the hard core is independent of parity, but not necessarily the spin. The reason for this assumption is that the odd-parity scattering does not depend sensitively on the radius of a small hard core unless the potential is very singular. Thus the calculation depends on fourteen parameters.

Radial Schrödinger equation:

S = 0 (singlet)

S = 1 (triplett)

J = L+S is the conserved quantum number

Each channel has its own parameters

Most famous:

ANNALS OF PHYSICS: 50, 411–448 (1968)

Local Phenomenological Nucleon-Nucleon Potentials*†

RODERICK V. REID, JR.

Some details and Comments:

$$V^{\mathrm{OPEP}} = (g^2/12)\, mc^2 (m/M)^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12}\left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\right] e^{-x}/x$$

Potential has as physics input the pion exchange

•Parameters are fitted to data = observables available at the time roughly to laboratory kinetic energy 300 MeV

•Short and intermediate range parts of the potential are parameterized by Yukawa functions.

•In 1993 the Nijmegen group refitted the parameters to current data for a REID93 potential

NN-OnLine
15 years 1994 - 2009

http://nn-online.org
29 March 2012
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- Physics in Nijmegen

W3C XHTML 1.1

Nucleon-Nucleon phase shifts

Make a figure of phase shifts as a function of T_{lab}.

Choose at least one, upto five, of the following models	☐ PWA: PWA93 ☐ POT: ESC96 ☐ POT: NijmI ☐ POT: NijmII ☑ POT: Reid93 ☐ POT: Nijm93
Choose the interaction	○ proton-proton ● neutron-proton
Choose a maximum T_{lab} (between 10 and 350 MeV)	300
Give the phase	3p2
Plot options	● PostScript ○ PDF ○ PNG ● color ○ black/white

Start Reset

https://gwdac.phys.gwu.edu/

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Partial-Wave Analyses at GW
[See Instructions]
- Pion-Nucleon
- Pi-Pi-N
- Kaon(+)-Nucleon
- Nucleon-Nucleon
- Pion Photoproduction
- Pion Electroproduction
- Kaon Photoproduction
- Eta Photoproduction
- Eta-Prime Photoproduction
- Pion-Deuteron (elastic)
- Pion-Deuteron to Proton+Proton

Analyses From Other Sites
- Mainz (MAID – Analyses)
- Nijmegen (Nucleon-Nucleon OnLine)
- Bonn-Gatchina (PWA)
- Juelich-Bonn-Washington (PWA)
- Joint Physics Analysis Center

Contact
- William Briscoe
- Michael Doering
- Helmut Haberzettl
- Igor Strakovsky
- Ron Workman

INS DAC Services [SAID Program]

- The SAID Partial-Wave Analysis Facility is based at GWU.
- New features are being added and will first appear at this site. Suggestions for improvements are always welcome.

Instructions for Using the Partial-Wave Analyses

The programs accessible with the left-hand side navigation bar allow the user to access a number of features available through the SAID program. Contact a member of our group if you are unfamiliar with the SSH version. If you enter choices which are unphysical, you may still get an answer (in accordance with the `garbage in, garbage out' rule). Please report unexpected garbage-out to the management.

Note: These programs use HTML forms to run the SAID code. If unfamiliar with the options, run the default setup first. The output is an (edited) echo of an interactive session which would have resulted had you used the SSH version. If the default example fails to clarify the specific task you have in mind, we can help (just send an e-mail message).

All programs expect energies in **MeV** units. All of the solutions and potentials have limited ranges of validity. Some are unstable beyond their upper energy limits. Extrapolated results may not make much sense.
Increments: The programs will not allow an arbitrary number of points to be generated. As a rule, stay below **50**.

ACKNOWLEDGMENTS

The **INS Data Analysis Center** is partially funded by the U.S. Department of Energy, and the Research Enhancement Funds of The George Washington University, with strong support from the GW Virginia Science and Technology Campus and technical support from Dr. G.A. MacLachlan.

Go to Top

1960's: Golden Era of Particle Physics

- Discovery of mesons and baryons
- Attempt: Mesons heavier than the pion responsible for the short range part of the nuclear force
- Field theory and Feynman diagrams
- **Symmetry:**
- Lorentz invariance

Fermions			Bosons
Leptons and Quarks	Spin = $\frac{1}{2}$	Spin = 1*	Force Carrier Particles
Baryons (qqq)	Spin = $\frac{1}{2}$, $\frac{3}{2}, \frac{5}{2} \dots$	Spin = 0, 1, 2...	Mesons ($q\bar{q}$)

Covariant bilinear forms:

$$
\begin{aligned}
\Gamma^s &= \mathbf{1} \\
\Gamma^V_\mu &= \gamma_\mu \\
\Gamma^T_{\mu\nu} &= \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu] \\
\Gamma^A_\mu &= \gamma_5 \gamma_\mu \\
\Gamma^p &= i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5 \equiv \gamma^5
\end{aligned}
$$

Meson Exchange NN forces

1960's

Many pions = multi-pion resonances:

$\sigma(600)$, $\rho(770)$, $\omega(782)$ ...

One-Boson-Exchange Model

1970's

Refined Meson Theories

Sophisticated models for two-pion exchange:

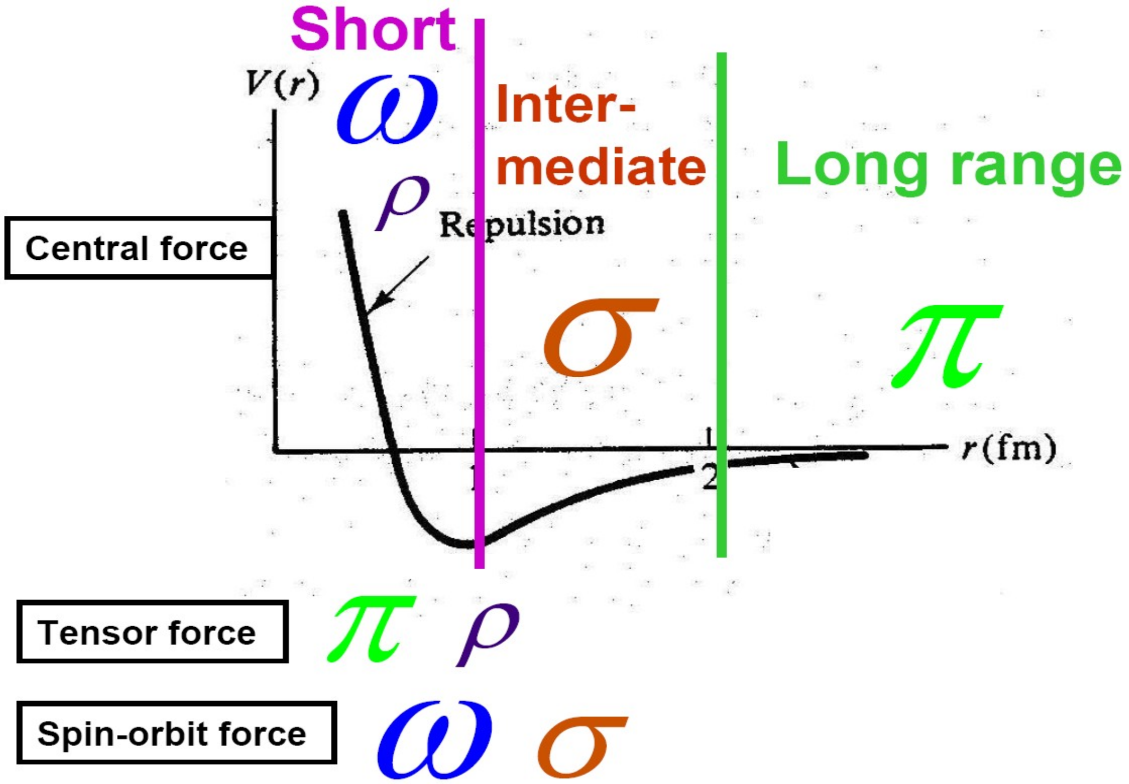
Paris Potential (Lacombe *et al.*, PRC **21**, 861 (1980))

Bonn potential (Machleidt *et al.*, Phys. Rep. **149**, 1 (1987))

Open Questions:

Which mesons and nucleon resonances ?
Loop diagrams of strong interaction diverge
=> **Cutoff's at vertices as effective size of the nucleon.**

Mesons in the NN Force

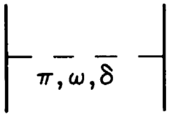


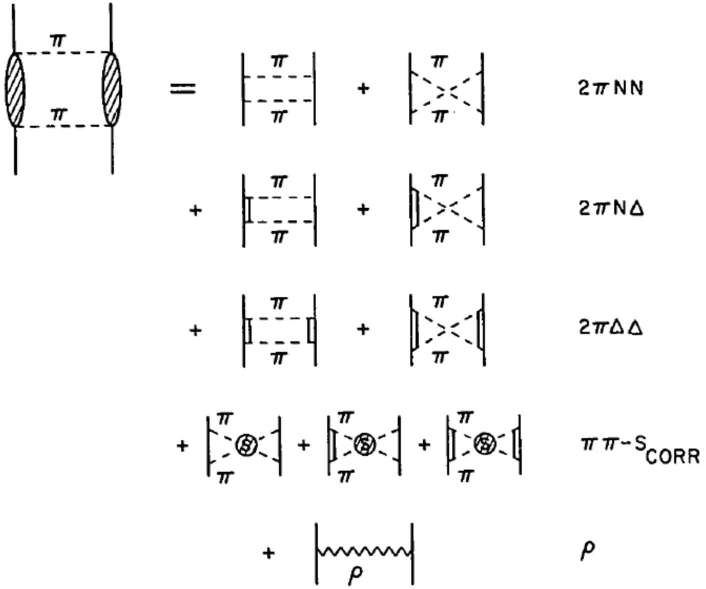
Meson diagrams of the Bonn Potential

R. Machleidt, K. Holinde, Ch. Elster

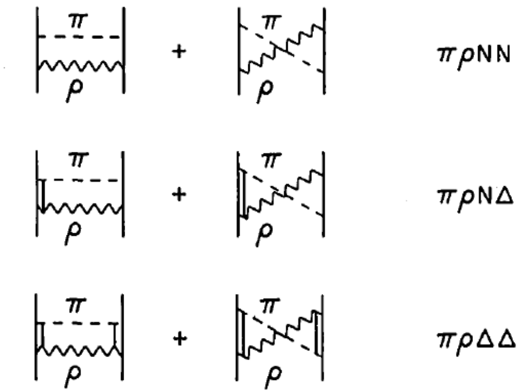
Phys.Rept. 149 (1987) 1-89

One boson exchange





2π exchange



3π exchange

physics + astronomy

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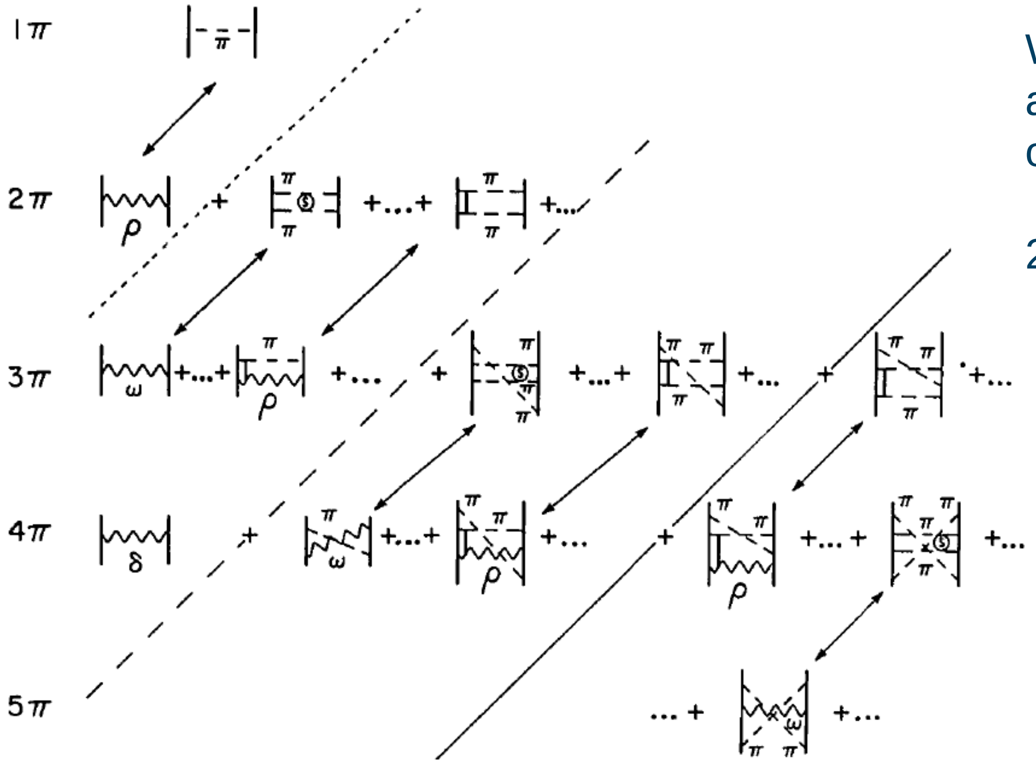
Meson diagrams of the Bonn potential organized according to pion exchanges



We could see convergence in the phase shifts and observableswhen using a specific grouping of the dagrams:
Always π and ρ (tensor) and
2π and 3π (ω) considered together

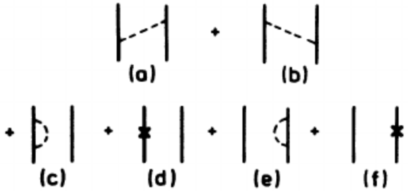
This is **not** a mathematical proof of convergence!

Explicit pions: Pion production in NN scattering above the pion production threshold (E_{lab}=287 MeV)

PHYSICAL REVIEW C VOLUME 37, NUMBER 4 APRIL 1988

Extension of the Bonn meson exchange NN potential above pion production threshold: Nucleon renormalization and unitarity

Ch. Elster, W. Ferchländer, K. Holinde,* and D. Schütte

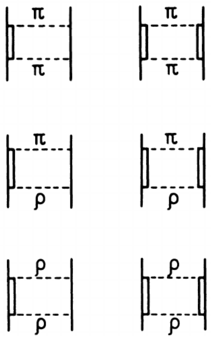


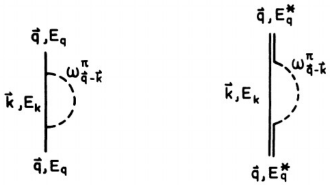
Pion self-energy diagrams need to be explicitly considered to preserve three-body unitarity.

PHYSICAL REVIEW C VOLUME 38, NUMBER 4 OCTOBER 1988

Extension of the Bonn meson exchange *NN* potential above pion production threshold: Role of the delta isobar

Ch. Elster,* K. Holinde,† and D. Schütte





Similar work in the same spirit: Different formalism

PHYSICAL REVIEW C VOLUME 33, NUMBER 6 JUNE 1986

Relativistic three-body approach to NN scattering at intermediate energies

E. van Faassen
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

J. A. Tjon
Institute for Theoretical Physics, 3508 TA Utrecht, The Netherlands

(Received 20 January 1986)

PHYSICAL REVIEW C VOLUME 36, NUMBER 4 OCTOBER 1987

Unitary meson-exchange πNN models: NN and πd elastic scattering

T.-S. H. Lee
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

A. Matsuyama*
Swiss Institute for Nuclear Research, Villigen, Switzerland and Faculty of Liberal Arts, Shizuoka University, Shizuoka, Japan

Unique up to today :
NN interaction up to 1 GeV

Progress in Three-Nucleon Physics asked for more accurate NN forces as input

1980's

Nijmegen: We need more precision!!!

"A χ^2/dat of ≈ 2 is not good enough, it has to be 1.0"

1990's

1993: The high-precision Nijmegen phase shift analysis

1994-2001: High-precision NN potentials:

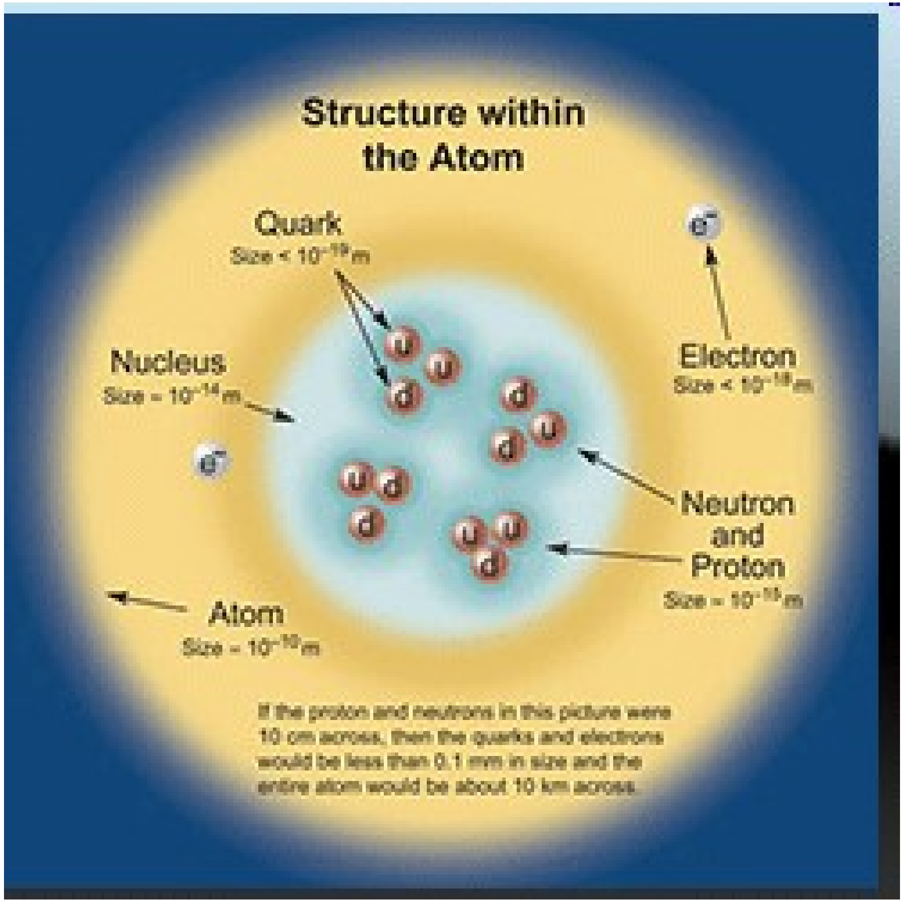
Nijmegen I, II, '93, Reid93 (Stoks et al. 1994)

Argonne V18 (Wiringa et al, 1995)

CD-Bonn (Machleidt et al. 1996, 2001)

Nijmegen and **CD-Bonn**: Partial waves fitted

AV18: Operators with scalar functions



1963 Gell-Mann: 3 quarks

1970's

Interaction between quarks via gluon exchange:

Quantum-Chromo-Dynamics (QCD)

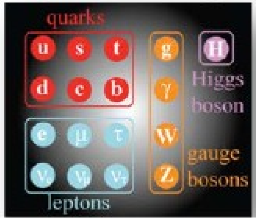
Effective Theory and Chiral Potentials

Degrees of freedom

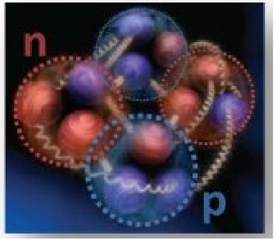
Quark/Gluon dynamics (QCD)

$$\mathcal{L}_{QCD} = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M} q_R$$

High energy



Low energy



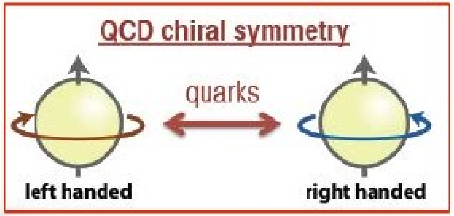
Nucleon/Pion dynamics

Tool: Effective Field Theory (EFT)

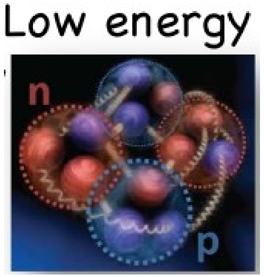
Symmetry

What is most important for a theory? The symmetries and not the degrees of freedom.

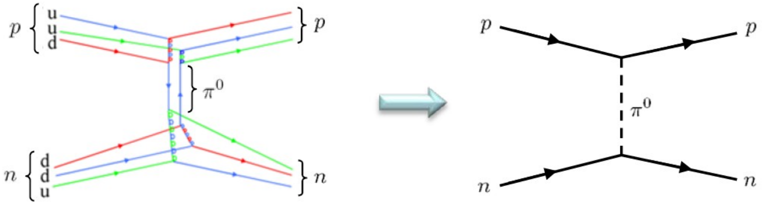
The usual (Lorentz covariance, parity, etc.)+
Chiral symmetry



Effective Theory and Chiral Potentials

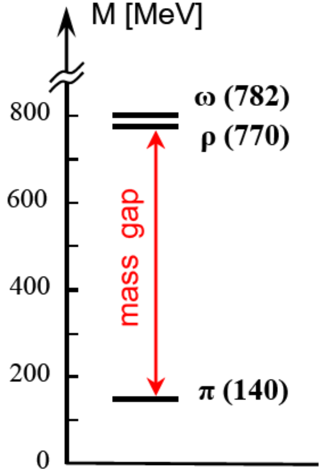


- Asymptotically observed states are effective degrees of freedom → EFT



- Spontaneously broken approximate chiral symmetry of QCD plays important role

- Light (m_π) and heavy (m_ρ) mass scales are well separated



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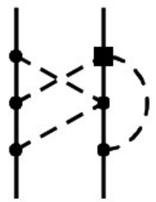
Chiral EFT for Nuclear Forces

- **Framework**
  - Use ChPT to calculate irreducible contributions (=nuclear force)
  - Solve Schrödinger equation to calculate observables

- **Power Counting**

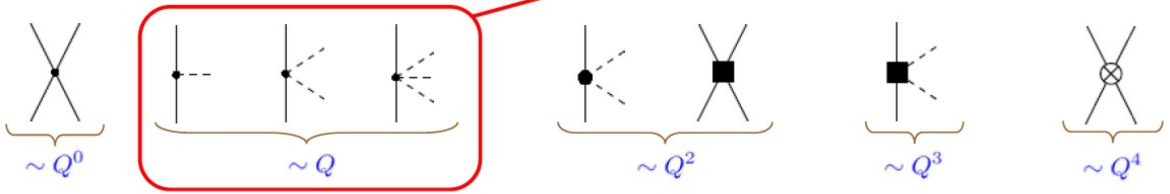


$$\sim \left(\frac{Q}{\Lambda}\right)^{\nu} \qquad \Lambda \sim M_\rho$$

- **Vertices**

constrained by chiral symmetry

Short range contact term ⇨



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Write down the most general Lagangian consistent with Symmetries
Hirarchie of terms →Power Counting

$$\mathcal{L}_{\text{eft}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

$$\begin{aligned}
\mathcal{L}^{(0)} &= \frac{1}{2}\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi} - \frac{1}{2}m_\pi^2 \boldsymbol{\pi}^2 + N^\dagger\left[i\partial_0 + \frac{g_A}{2f_\pi}\boldsymbol{\tau}\sigma\cdot\nabla\boldsymbol{\pi} - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})\right]N \\
&\quad - \frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\sigma N)(N^\dagger\sigma N) + \ldots, \\
\mathcal{L}^{(1)} &= N^\dagger\left[4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\boldsymbol{\pi}^2 + \frac{c_2}{f_\pi^2}\dot{\boldsymbol{\pi}}^2 + \frac{c_3}{f_\pi^2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})\right. \\
&\quad \left. - \frac{c_4}{2f_\pi^2}\epsilon_{ijk}\,\epsilon_{abc}\,\sigma_i\tau_a(\nabla_j\,\pi_b)(\nabla_k\,\pi_c)\right]N \\
&\quad - \frac{D}{4f_\pi}(N^\dagger N)(N^\dagger\sigma\boldsymbol{\tau}N)\cdot\nabla\boldsymbol{\pi} - \frac{1}{2}E\,(N^\dagger N)(N^\dagger\boldsymbol{\tau}N)\cdot(N^\dagger\boldsymbol{\tau}N) + \ldots
\end{aligned}$$

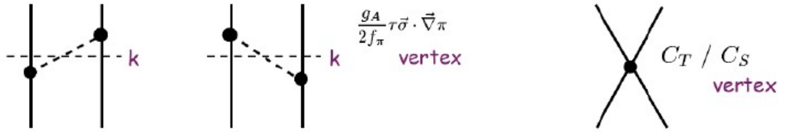
Weinberg counting

Infinite # of unknown parameters (LEC's), but leads to hierarchy of diagrams: $\nu = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2) \geq 0$

N = # external nucleons
L = # loops
d_i = # derivatives or m_π at i^{th} vertex
n_i = # nucleons at i^{th} vertex

Calculate to the desired order

LO time-ordered diagrams



$$V_{1\pi}(\mathbf{q}) = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{\tau_1\cdot\tau_2}{2\omega_q}\left\{\frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\frac{\mathbf{p}_1^2}{2M}+\frac{\mathbf{p}_2^2}{2M}-\frac{\mathbf{p}_1'^2}{2M}-\frac{\mathbf{p}_2^2}{2M}-\omega_q}+\text{2nd diagram}\right\}$$
$$= -\left(\frac{g_A}{2f_\pi}\right)^2 \tau_1\cdot\tau_2\frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}$$

one pion exchange

zero-range contact term at LO

$$V_C = C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

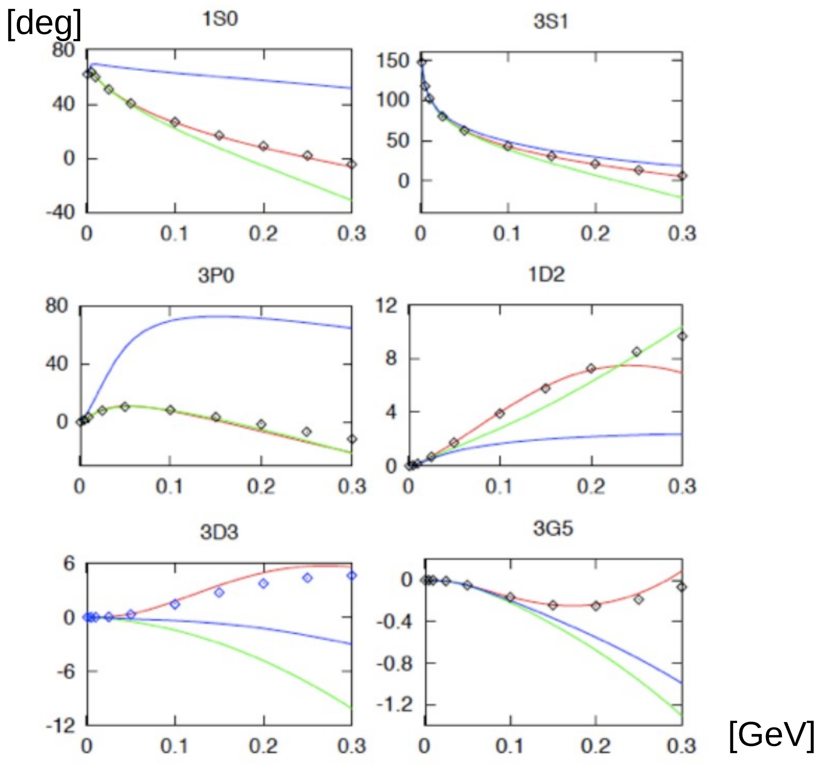
regularize (WHY?)

$$V(\mathbf{p}',\mathbf{p}) \rightarrow e^{-(p'/\Lambda)^{2n}}V(\mathbf{p}',\mathbf{p})e^{-(p/\Lambda)^{2n}}$$

Chiral EFT for the two-nucleon potential

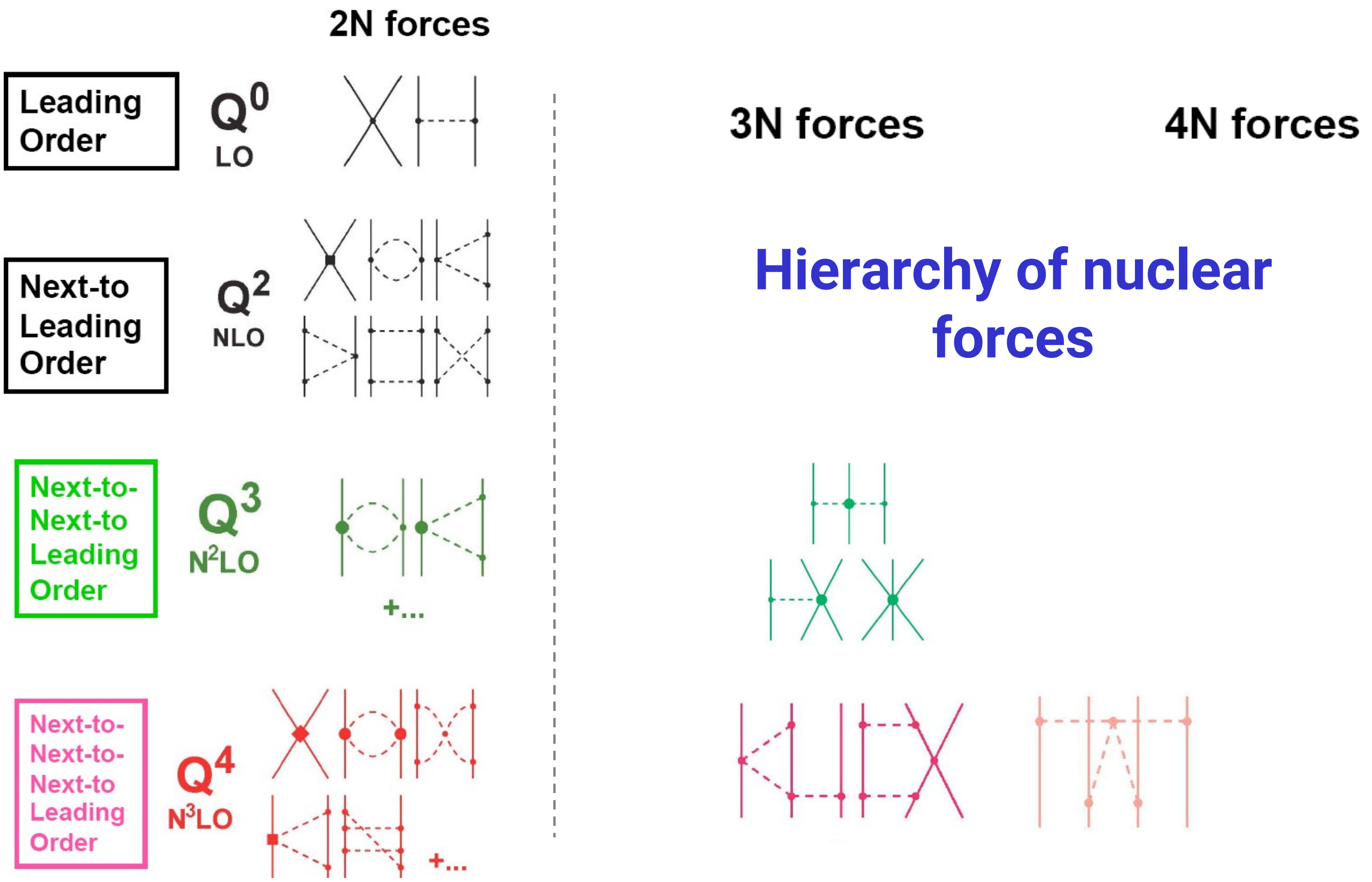
- Epelbaum, Meißner, et al.
- Also Entem, Machleidt
- $\mathcal{L}_{\pi N}$ + match at low energy

Q^ν	1π	2π	$4N$
Q^0		—	(2)
Q^1			
Q^2			$\vec{\nabla}^2$ (7)
Q^3	$\mathcal{L}_{\pi N}$	$\mathcal{L}_{\pi N}$	
Q^4	many	many	$\vec{\nabla}^4$ (15)



Approaches level of accuracy (and fit parameters via the LECs) of "conventional" models at N3LO

Hierarchy of nuclear forces



Why the cutoff Λ ?

Need to match unknown LECs to data (e.g., phaseshifts). Solve LS eqn:

$$T(k,k) = V(k,k) + \frac{2}{\pi}\int q^2 dq \frac{V(k,q)T(q,k)}{k^2 - q^2} \quad \text{where} \quad \tan\ (k) = -kT(k,k)$$

NN loop integral UV divergent => regularization and renormalization

- details of cutoff (sharp, smooth, etc.) don't matter to low E physics
- LECs now "run" with Λ

No such thing as "the" chiral potential of a given order. Infinitely many regularization/renormalization schemes => any differences should be higher order effects.

Truncation errors of observables go as $\mathcal{O}(\frac{Q^\nu}{\Lambda^\nu})$

"theoretical error bars" from varying Λ

From Epelbaum, Krebs, Meissner, *Eur. Phys. J. A51*, 53 (2015)

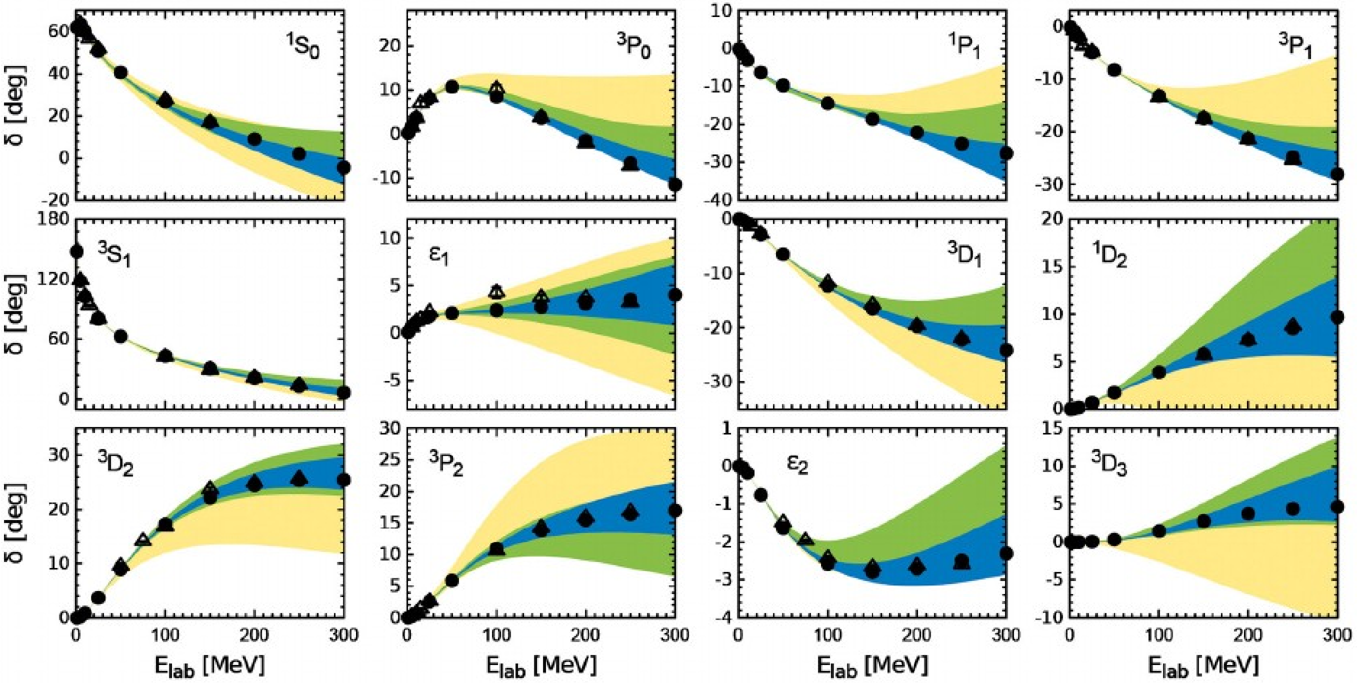


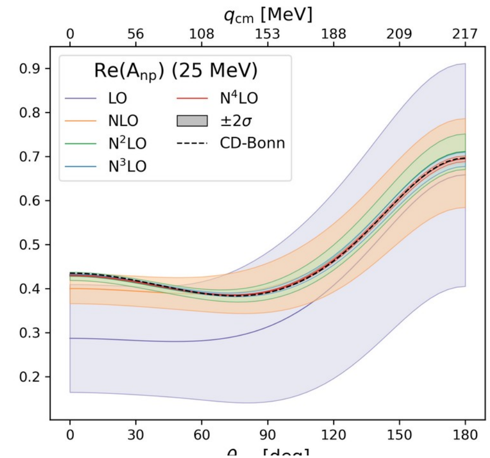
Fig. 9. (Color online) Estimated theoretical uncertainty of the np phase shifts at NLO, N^2LO and N^3LO based on the cutoff of $R = 0.9$ fm in comparison with the NPWA [45] (solid dots) and the GWU single-energy np partial wave analysis [94] (open triangles). The light-(yellow), medium-(green) and dark-(blue) shaded bands depict the estimated theoretical uncertainties at NLO, N^2LO and N^3LO, as explained in the text. Only those partial waves are shown which have been used in the fits at N^3LO.

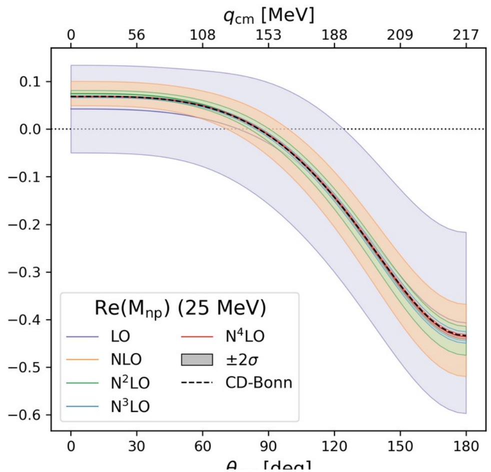
Order-by-order convergence of Wolfenstein np amplitudes:

EKM chiral potential with cutoff R=0.9 fm

A ~ **1**

Central part of interaction

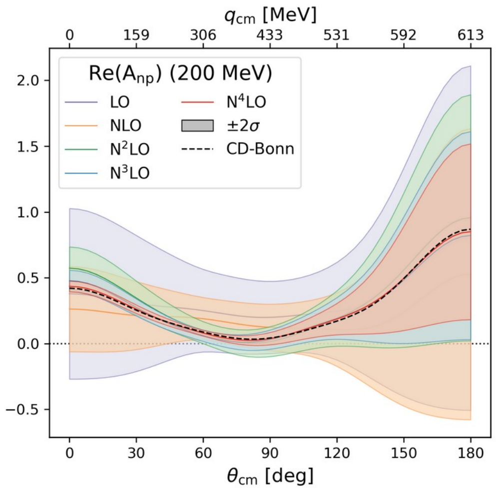


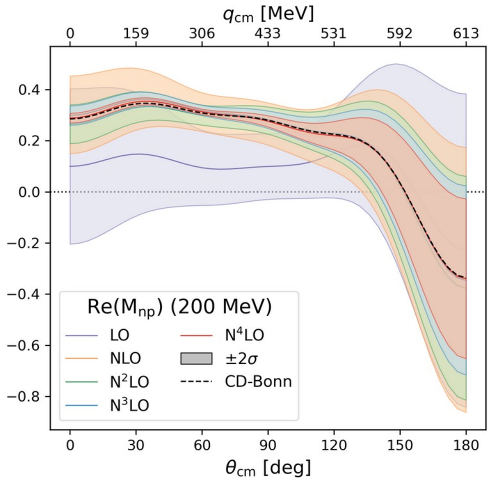


M ~ tensor part of interaction

Λ_b=750 MeV

Found by doing Bayesian statistical analysis





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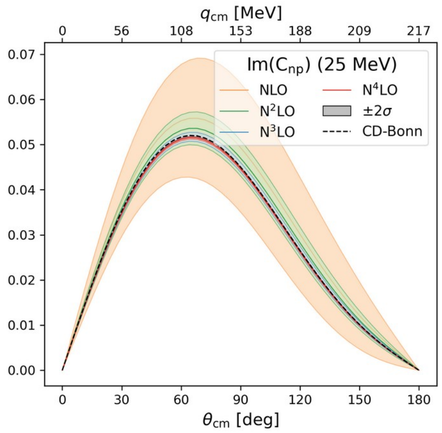
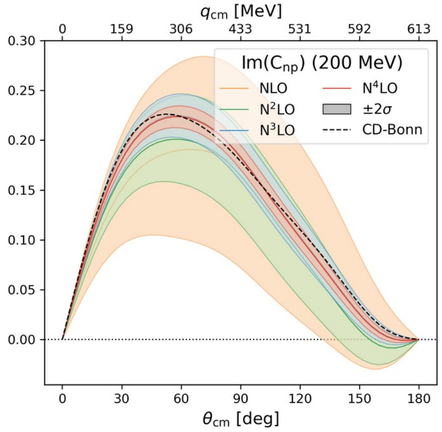
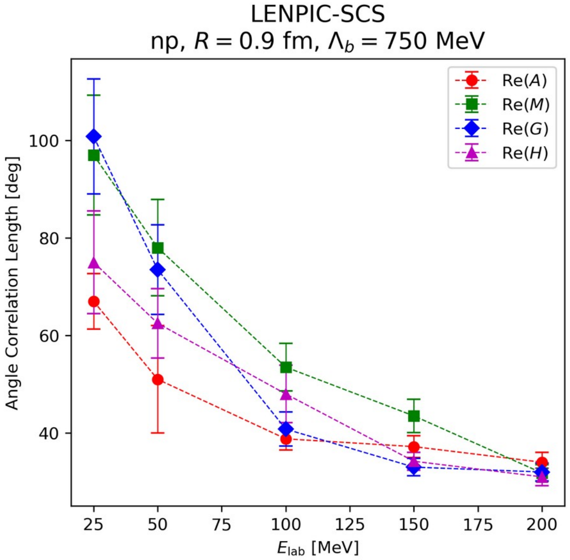
Order-by-order convergence of Wolfenstein np amplitudes:
EKM chiral potential with cutoff R=0.9 fm

Im C ~ ***i*σ·n**

Spin-Orbit part of potential







Correlation length as function of angle

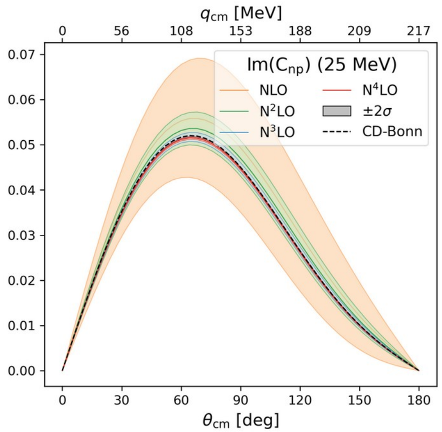
B. McClung,
D.R. Phillips,
Ch. Elster

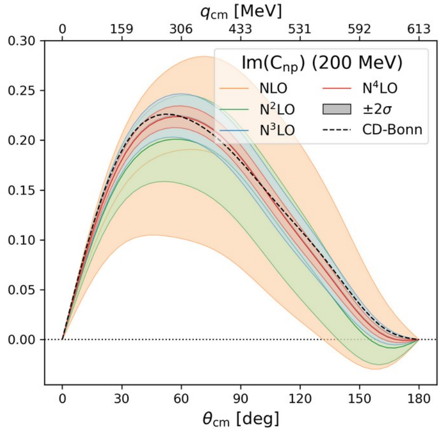
Order-by-order convergence of Wolfenstein np amplitudes:
EKM chiral potential with cutoff R=0.9 fm

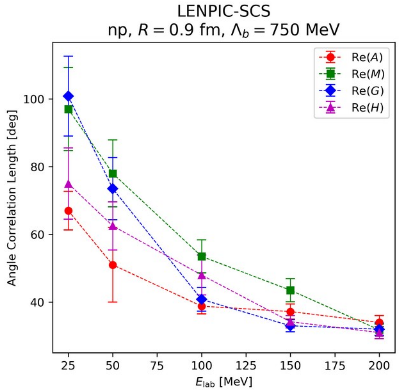
B. McClung,
D.R. Phillips,
Ch. Elster

Im C ~ $i\boldsymbol{\sigma}\cdot\mathbf{n}$

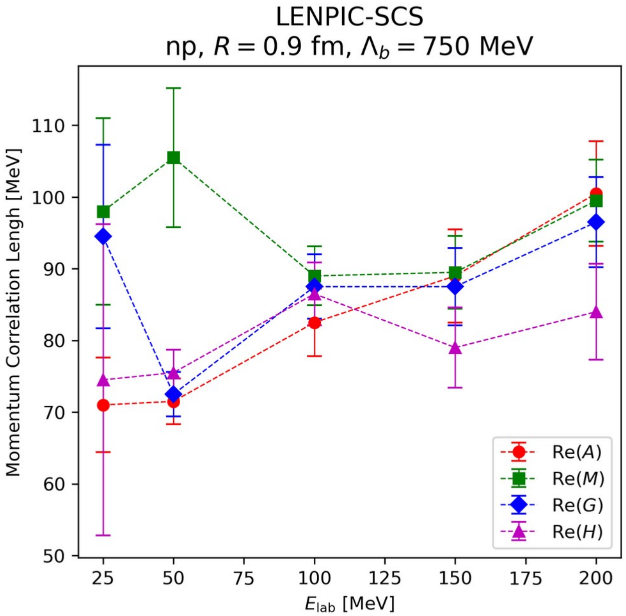
Spin-Orbit part of potential







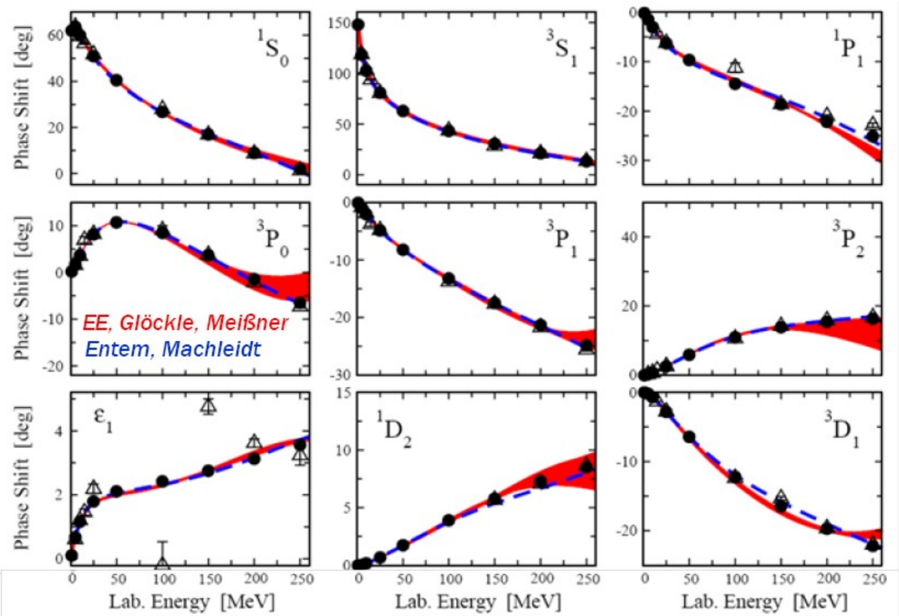
Correlation length as **function of momentum** is relatively **constant** with respect to
- **Lab energies**
- **amplitudes**



Chiral two-nucleon forces: Summary

- Contact terms are fixed by observables
- In practice: different partial waves fixed different contact terms
- The higher the order, the better the description of the phase shift.

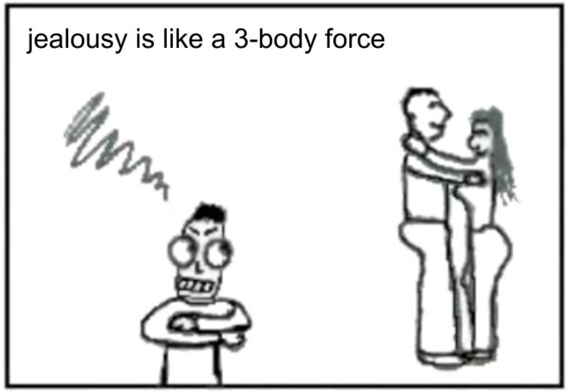
Ordonez, Ray, van Kolck.'94,'96
Kaiser, Brockmann, Weise '97
E.E., Glöckle, Meißner '98 - '05
Kaiser '99 - '02
Higa, Robilotta, da Rocha '03 - '05
Entem, Machleidt '02 - '04

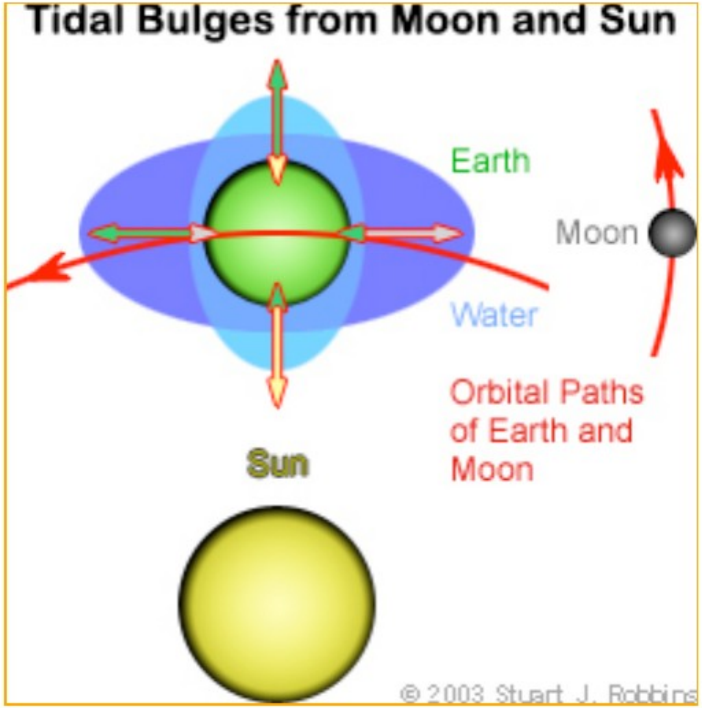


Three-body Forces

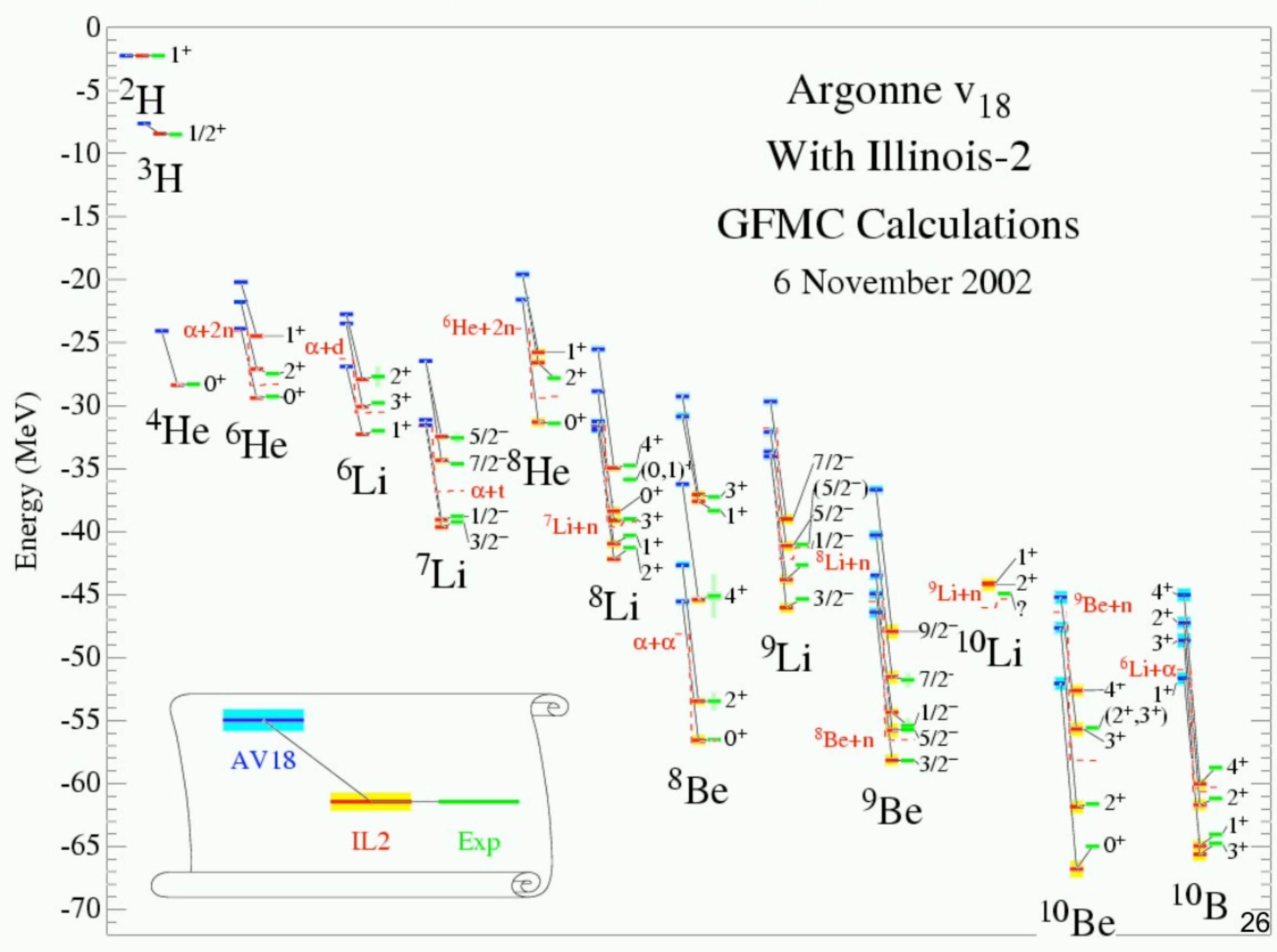
From Wikipedia, the free encyclopedia

A **three-body force** is a force that does not exist in a system of two objects but appears in a three-body system. In general, if the behaviour of a system of more than two objects cannot be described by the two-body interactions between all possible pairs, as a first approximation, the deviation is mainly due to a three-body force.





Evidence for 3N in light nuclei: overall binding & level ordering



A theorem for three-body Hamiltonians

Polyzou and Glöckle, Few Body Systems 9, 97 (1990)

Different two-body Hamiltonians can be made to fit two-body and three-body data by including a 3NF into one of the Hamiltonians

Theorem. *Let*

$$H_{ij} = H_i + H_j + V_{ij} \quad \text{and} \quad \bar{H}_{ij} = H_i + H_j + \bar{V}_{ij} \tag{1.1}$$

be two-body Hamiltonians with the same binding energies and scattering matrices for each pair of particles i and j. Assume that the two-body Hamiltonians are asymptotically complete and that the unitary transformations relating these two-body Hamiltonians, which necessarily exist, have bounded Cayley transforms. Then there exists a three-body interaction, W, such that the two three-body Hamiltonians

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} \tag{1.2}$$

and

$$\bar{H}' = \bar{H} + W \tag{1.3}$$

with

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} \tag{1.4}$$

have the same binding energies and scattering matrix.

Corollary. *Under the assumptions of the theorem, if $V_{(123)}$ is a three-body interaction then there exists another three-body interaction $\bar{V}_{(123)}$ such that*

$$H = H_1 + H_2 + H_3 + V_{12} + V_{23} + V_{31} + V_{(123)}$$

and

$$\bar{H} = H_1 + H_2 + H_3 + \bar{V}_{12} + \bar{V}_{23} + \bar{V}_{31} + \bar{V}_{(123)}$$

have the same binding energies and scattering matrix.

Implications:

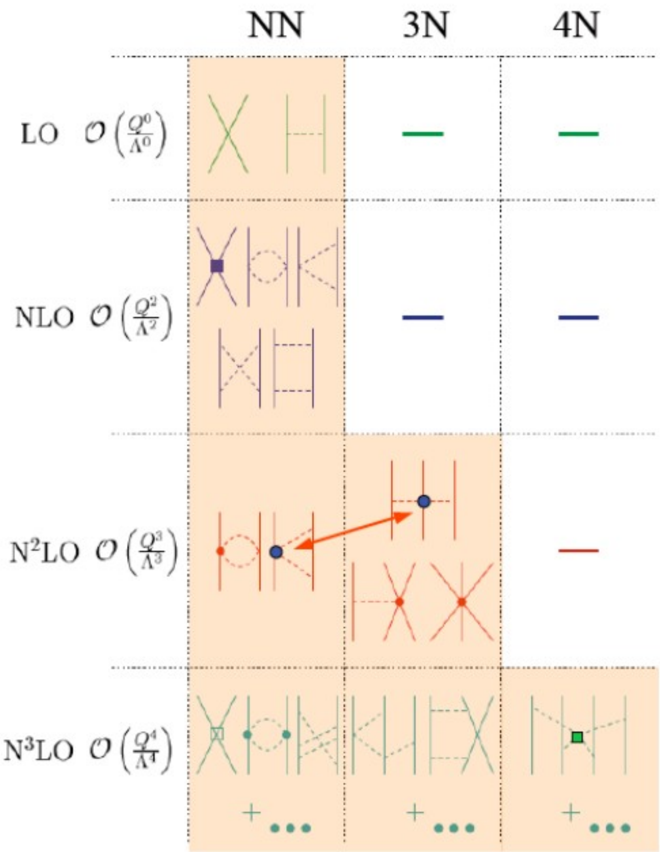
(1) There are no experiments measuring only three-body binding energies and phase shifts that can determine if there are no three-body forces in a three-body system. The question makes no sense. The correct statement is that there may be some systems for which it is possible to find a representation in which three-body forces are not needed.

(2) Different off-shell extensions of two-body forces can be equivalently realized as three-body interactions.

(4) Three-body forces cannot be determined in a manner that is independent of the two-body interaction.

Few-Body Forces from Chiral EFT

Separation of scales: low momenta Q << Λ_b breakdown scale



Explains 2N > 3N > 4N

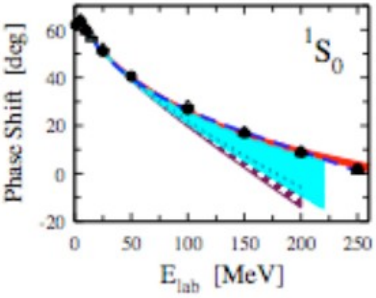
Formal Consistency

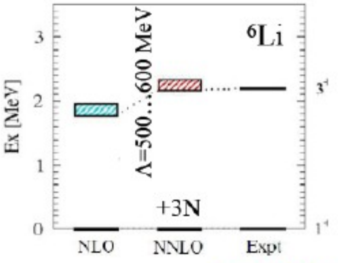
NN and NNN from same Lagrangian

ππ and πN, electroweak

Broken chiral symmetry of QCD

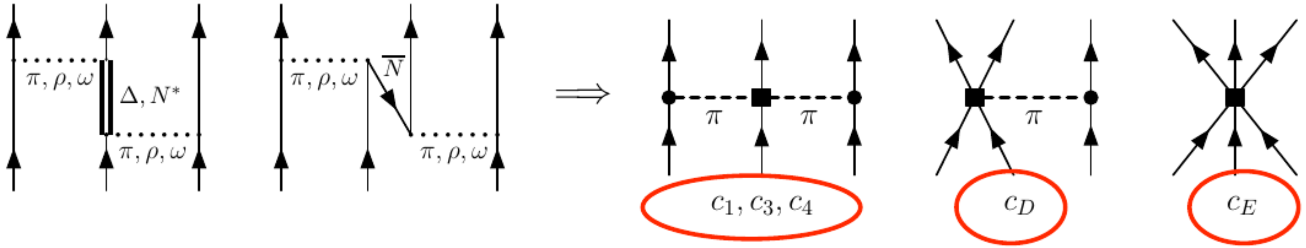
Error estimates from Λ variation





from A. Nogga

Eliminating DOF leads to 3-body forces



Leading three-nucleon force

1. Long-ranged two-pion term (Fujita & Miza …)
2. Intermediate-ranged one-poin term
3. Short-ranged three-nucleon contact

The question is not: Do three-body forces enter the description?

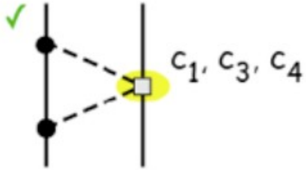
The (only) question is: How large are three-body forces?

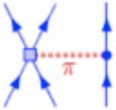
And at what resolution scale?

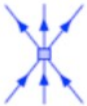
Typical fitting of 3NF LEC's at N^{2}LO (A. Nogga)

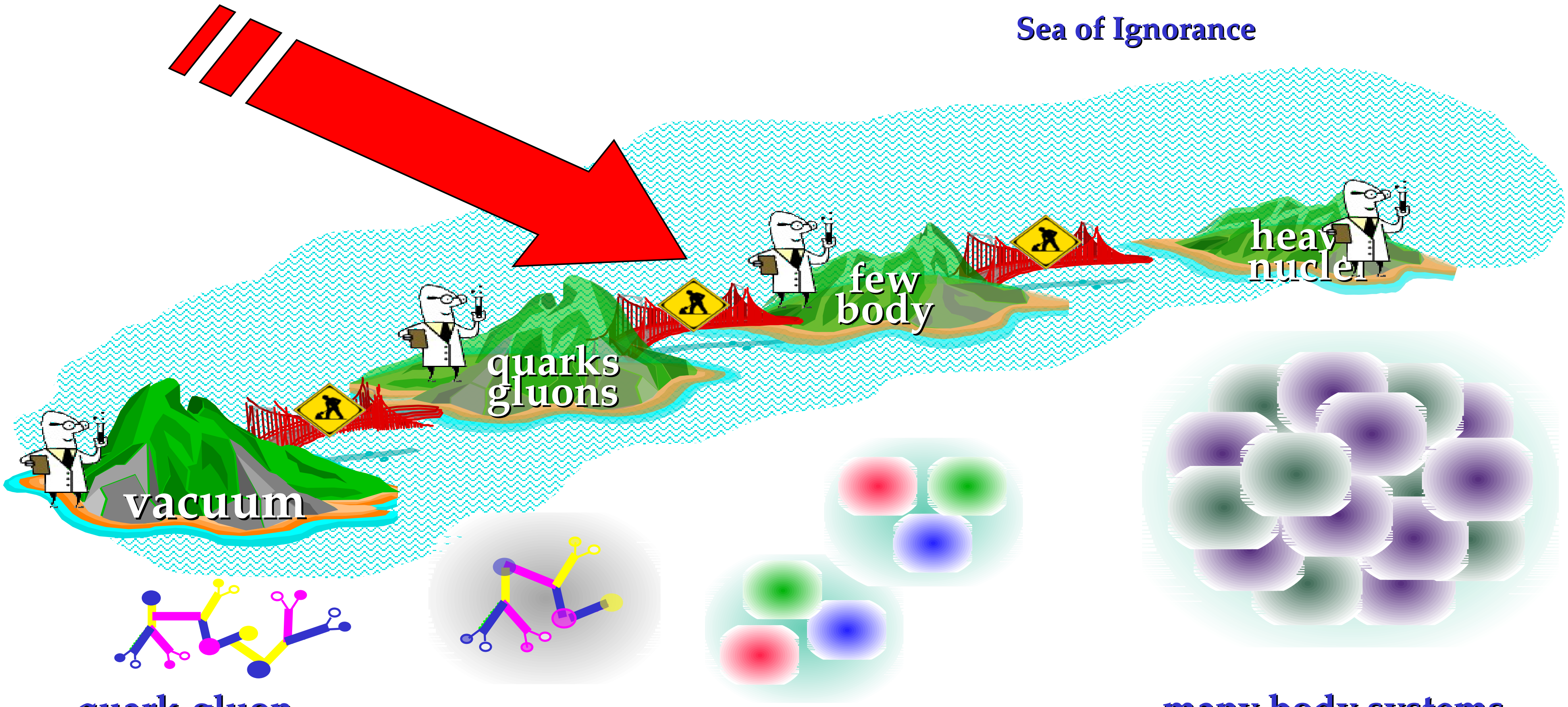
- Fitting c_1, c_3, and c_4

	c_1	c_3	c_4
NN phase shift analysis	-0.76	-4.78	3.96
πN scattering (dispersion rel.)	-0.81	-4.70	3.40
πN scattering (directly)	-1.23	-5.94	3.47
NN pert. 3F4	-0.81	-3.40	3.40
NN potential fit to data	-0.81	-3.20	5.40



- Significant uncertainties!
- Fitting *D* 

and *E* 
  - *D* appears in pion production from NN, but not analyzed
  - *E* requires a 3N observable
  - Typically *D* and *E* fit together to triton binding energy and ^{4}He binding energy *or* radius; or sometimes to 3-body energy and scattering length



quark-gluon soup
QCD

nucleon
QCD

few body systems
free NN force

many body systems
effective NN force